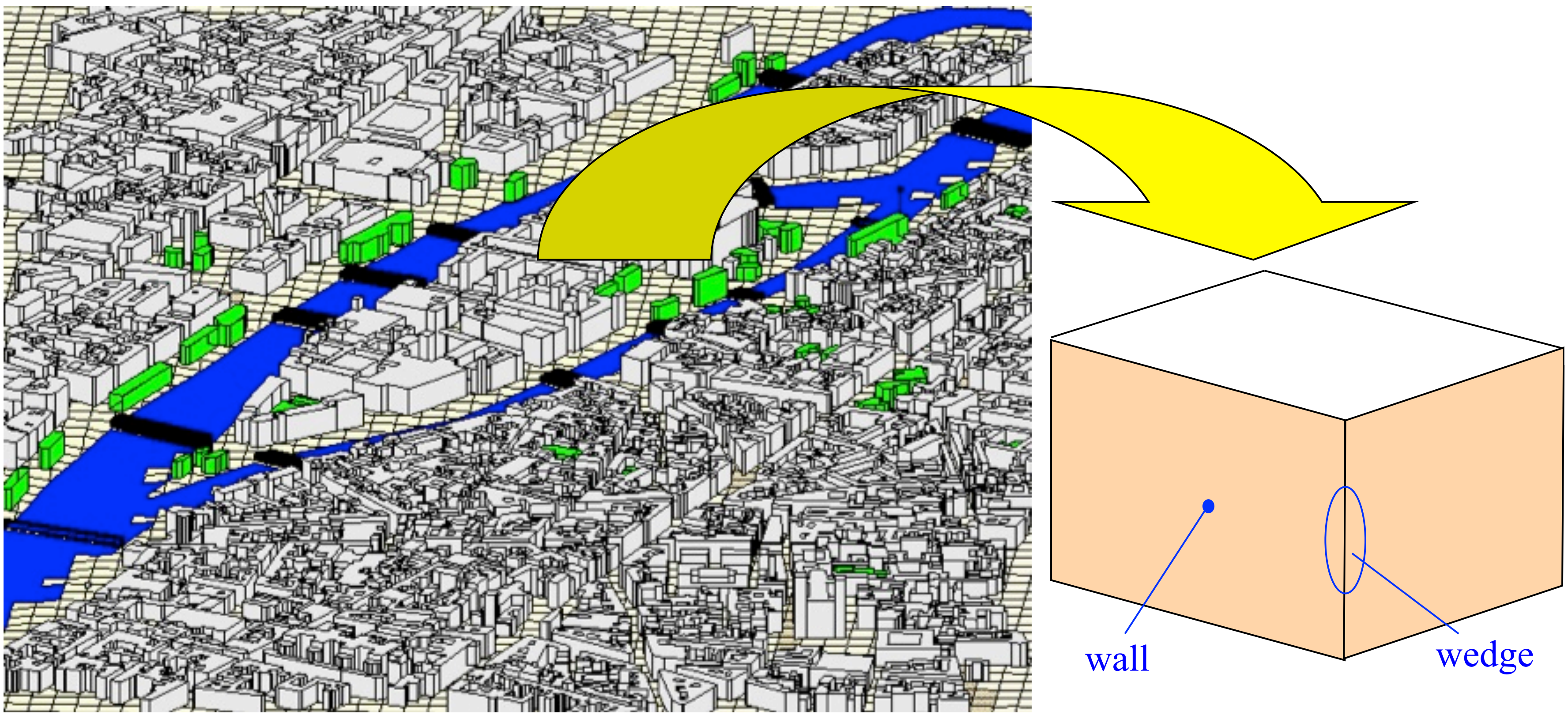
A - TEORIA DELLA PROPAGAZIONE RADIO IN AMBIENTE REALE

- Effetto di gas atmosferici e idrometeore
  - Attenuazione supplementare da gas atmosferici
  - Attenuazione supplementare da pioggia
  - Propagazione ionosferica, troposcatter
- Propagazione in mezzi con disomogenità distribuita – Propagazione troposferica
  - Cenni di ottica geometrica in mezzi con n debolmente variabile.
  - Propagazione in mezzi a stratificazione piana e sferica. Propagazione troposferica, orizzonte radio e rettificazione del suolo/raggio
- Propagazione in mezzi con disomogenità concentrate – Propagazione in presenza di ostacoli
  - Riflessione del suolo, diffrazione da knife-edge, ellissoide di Fresnel
  - Metodi per il calcolo della diffrazione da ostacoli
  - Teoria geometrica della propagazione: trasmissione attraverso uno strato, diffrazione da spigolo. Propagazione multicammino.

Geometrical theory of propagation (I)

It is useful when propagation takes place in a region with <u>concentrated obstacles</u>. Obstacles are here represented as <u>plane walls</u> and <u>rectilinear wedges</u> (*canonical obstacles*)



Geometrical theory of propagation (II)

electromagnetic constants:

air	wall (generic medium)	
$\varepsilon_o = \frac{1}{36\pi} 10^{-9}\ Farad/m$	ε	electric permittivity
$\mu_o = 4\pi 10^{-7}\ Henry/m$	$\mu = \mu_o$	magnetic permeability
$\sigma = 0$	σ (if lossy)	electric conductivity
	$\varepsilon_c = \varepsilon + \frac{\sigma}{j\omega} = \varepsilon - j\frac{\sigma}{\omega}$	complex permittivity
n=1	$n \triangleq \sqrt{\frac{\varepsilon_c}{\varepsilon_o}}$	refraction index
$\eta_o = 120\pi\ \Omega$	$\eta \triangleq \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu_o}{\varepsilon_c}}$	intrinsic impedance

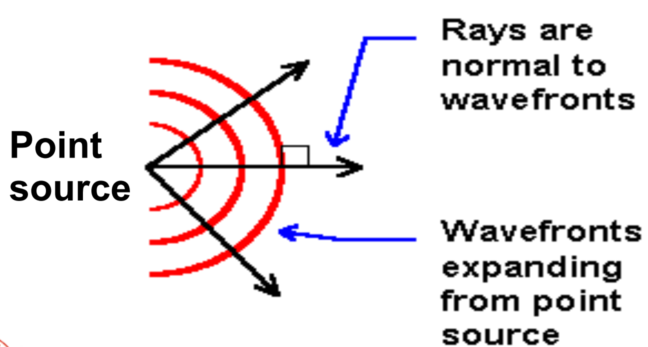
Geometrical theory of propagation (III)

- Geometrical theory of propagation is an extension of Geometrical Optics, (GO) and is not limited to optical frequencies $\left(\lambda \to 0 \text{ so that } \Delta n \to 0 \text{ over } \lambda\right)$
- Like GO, it corresponds to an asymptotic, high-frequency approximation of basic electromagnetic theory, and is based on the *ray concept*
- Since GO does not account for diffraction, then diffraction is introduced through an extension called Geometrical Theory of Diffraction (GTD)
- The combination of GO and GTD, applied to radio wave propagation may be called Geometrical Theory of Propagation (GTP) and is the base of deterministic, ray propagation models (ray-tracing etc.)

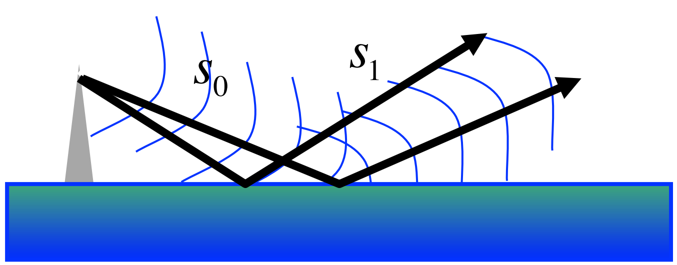
Recall: waves and rays

- Wavefront: surface were the field has the same phase (varies "in phase")
- Ray: given a propagating wave, every curve that is everywhere perpendicular to the wavefront is called ray. A ray is *the path* of a wave. There is a mutual identification btw wave and ray
- In presence of concentrated obstacles rays are piecewise-rectilinear and wavefronts can be of various kinds (see further on)

*Ex.1 Sperical wave and **rectilinear** rays*

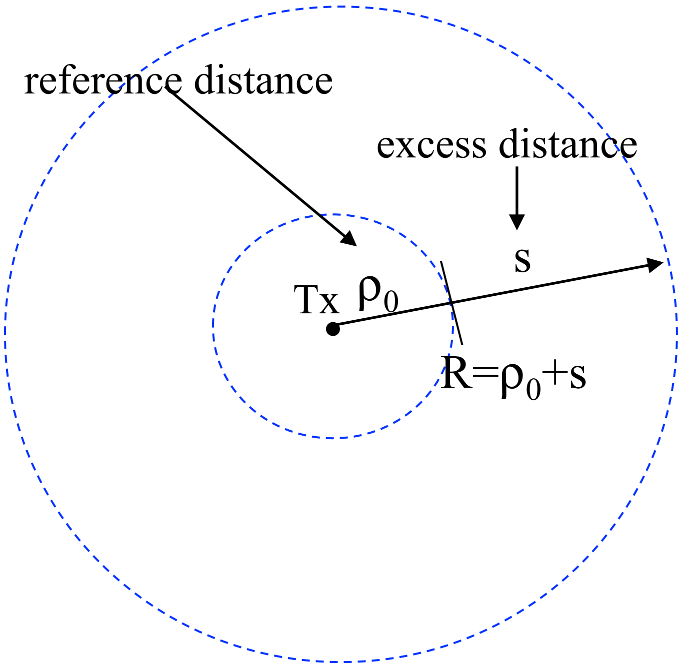


*Ex.2 reflected spherical wave and **piece-wise rectilinear** rays*



The spherical wave

In free space the reference wave is the spherical wave



field:

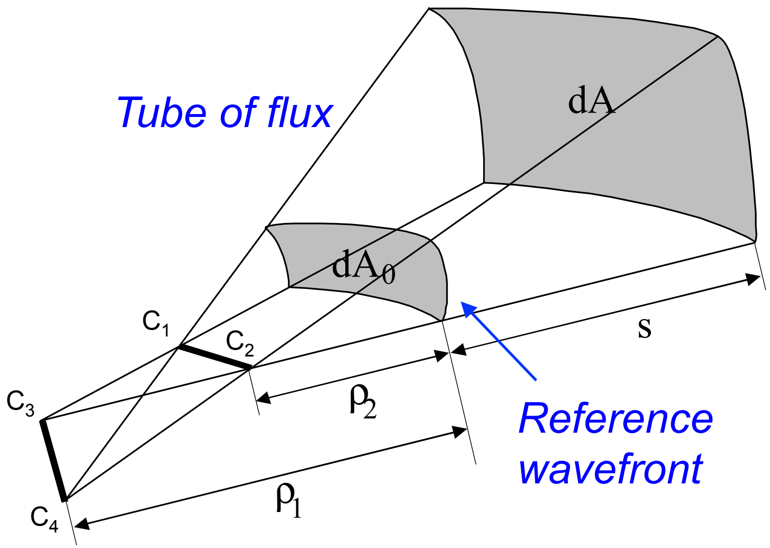
$$\mathbf{E}(R) = \mathbf{E}_0 \frac{e^{-j\beta R}}{R} = \mathbf{E}_0 \frac{e^{-j\beta\rho_0}}{\rho_0} \; \frac{\rho_0}{R} e^{-j\beta(R-\rho_0)} =$$

$$\mathbf{E}(\rho_0) \frac{\rho_0}{R} e^{-j\beta(R-\rho_0)} = \mathbf{E}(\rho_0) \frac{\rho_0}{\rho_0 + s} e^{-j\beta(s)}$$

power density:

$$S(R) = S(\rho_0) \left(\frac{\rho_0}{\rho_0 + s} \right)^2$$

The astigmatic wave: divergence factor



If the mean is homogeneous (➔ rectilinear rays) [2] the generic wave's divergence factor is:

$$A(\rho_1,\rho_2,s)=\sqrt{\frac{\rho_1\cdot\rho_2}{(\rho_1+s)\cdot(\rho_2+s)}}\left(=\sqrt{\frac{dA_0}{dA}}\right)$$

- ***A* : Divergence or Spreading factor**
- ρ_1, ρ_2 : curvature radii
- $\overline{C_1C_2}$, $\overline{C_3C_4}$: wave caustics

There are 3 main reference cases:

- Spherical wave: $\rho_1=\rho_2=\rho_0$ ➔ $A=\frac{\rho_0}{\rho_0+s}$
- Cylindrical wave: $\rho_1=\infty$, $\rho_2=\rho_0$ ➔ $A=\sqrt{\frac{\rho_0}{\rho_0+s}}$
- Plane wave: $\rho_1=\rho_2=\infty$ ➔ $A = 1$

notice that, for power conservation:

$$A=\sqrt{\frac{dA_0}{dA}}=\sqrt{\frac{|E|^2}{|E_0|^2}}=\frac{|E|}{|E_0|}=\sqrt{\frac{1}{L}}$$

L: power attenuation

The generic wave: amplitude and polarization

The divergence factor gives the field- (and thus power-) attenuation law along the ray. But since the field is a complex vector, we also have polarization.
The generic (astigmatic) wave in free space has the electric field:

$$\vec{E}(s) = \underbrace{\vec{E}(0)}_{\text{Field at reference point (s=0)}} \cdot \underbrace{\underbrace{\sqrt{\frac{\rho_1 \cdot \rho_2}{(\rho_1 + s)\cdot(\rho_2 + s)}}}_{\textbf{Divergence factor}} \cdot \underbrace{e^{-j\beta s}}_{\text{Phase factor}}}_{\text{Propagation factor}}$$

(Ex: spherical wave)

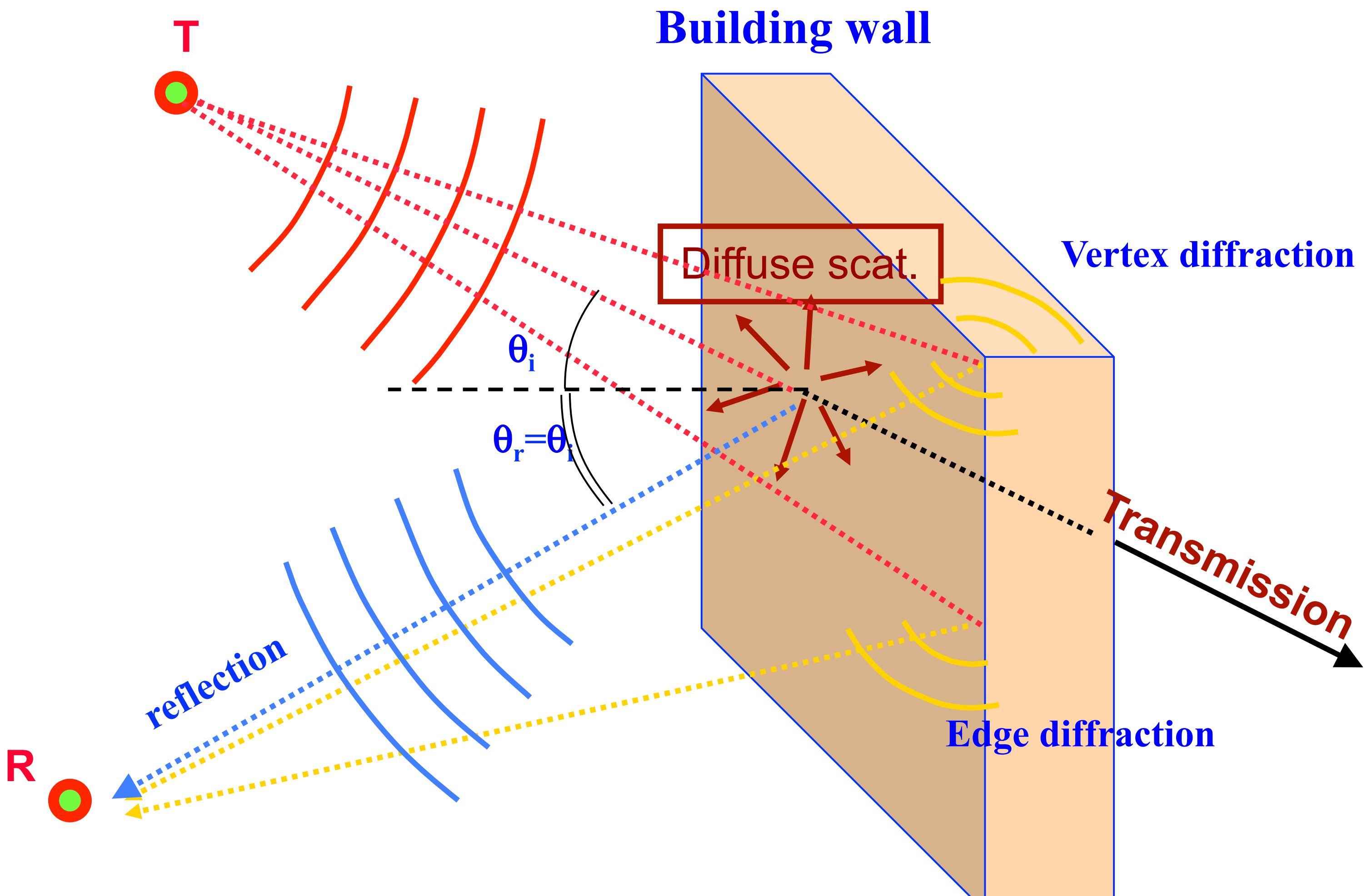
$$\vec{E}(s) = \vec{E}_0 \cdot \frac{\rho_0}{\rho_0 + s} e^{-j\beta s} = \hat{\mathbf{p}} \mathrm{K} \left(\frac{e^{-j\beta\rho_0}}{\rho_0} \right) \frac{\rho_0}{\rho_0 + s} e^{-j\beta s} = \hat{\mathbf{p}} \mathrm{K} \underbrace{\left(\frac{e^{-j\beta s_{tot}}}{s_{tot}} \right)}_{\text{Propagation factor}}$$

This expression gives the field amplitude *along* a ray.
The (normalized) ***polarization vector*** gives the polarization of the wave:

$$\hat{\mathbf{p}} \triangleq \frac{\vec{E}(s)}{\left|\vec{E}(s)\right|} e^{j\chi}$$

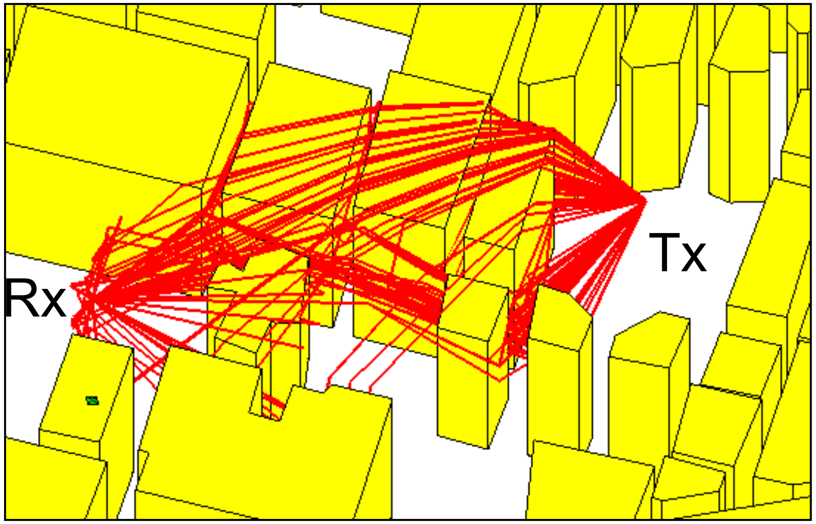
The polarization vector has the same polarization as the field but is normalized. In free space it is constant along the ray. The ***antenna polarization vector*** is the polarization vector of the field emitted by the considered antenna.

Interaction mechanisms



GTP basics

- GTP is based on the couple: (ray , field)
- The propagating field is computed as a set of rays interacting with building walls

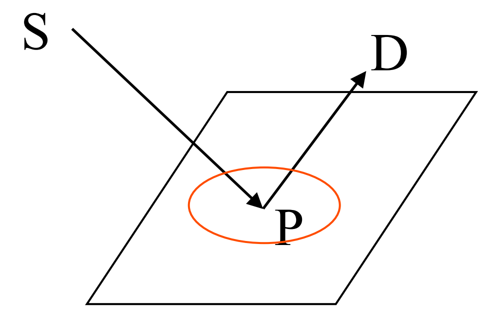


- Given a ray departing from an antenna we must "follow" the ray and predict both its geometry and its field at every point until it reaches the receiver
- It is therefore necessary to predict what happens at both the trajectory and the field at each interaction with an obstacle
- For this purpose, we rely on the two GTP basic principles

GTP: basic principles

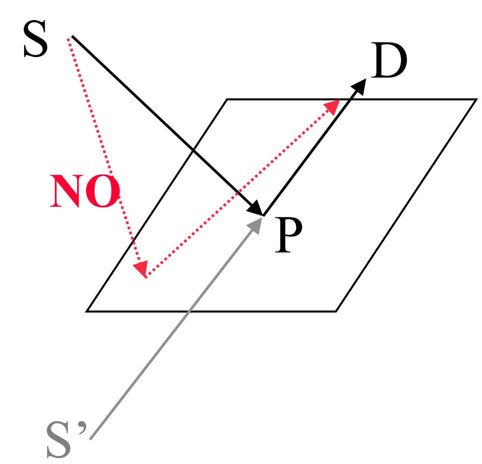
"Local field principle" (interactions)

- The wave can be locally assumed plane
- The field associated with the reflected/transmitted/ diffracted ray only depends on the electromagnetic and geometric properties of the obstacle in the vicinity of the interaction point

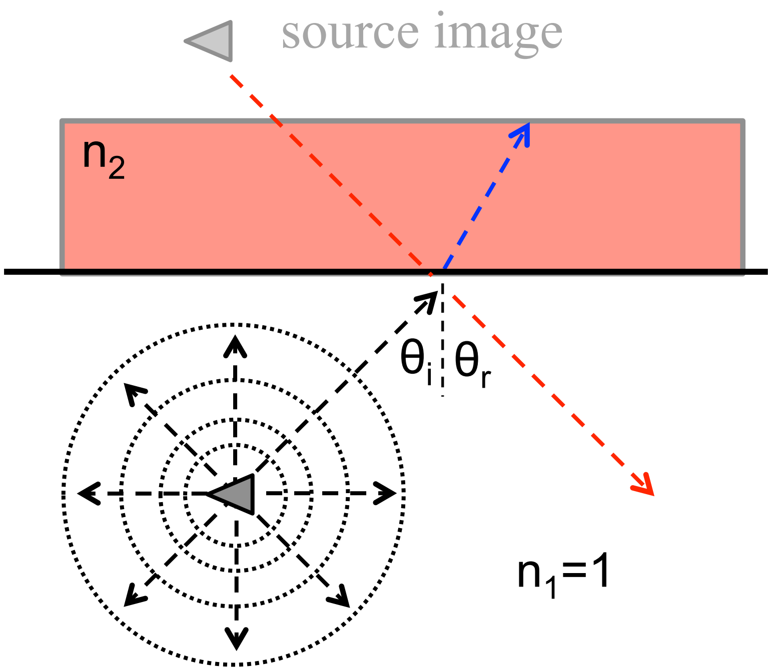


"Fermat's principle" (trajectory)

- The ray trajectory is always so as to minimize path (or optical-path …)



Ray Reflection and Transmission



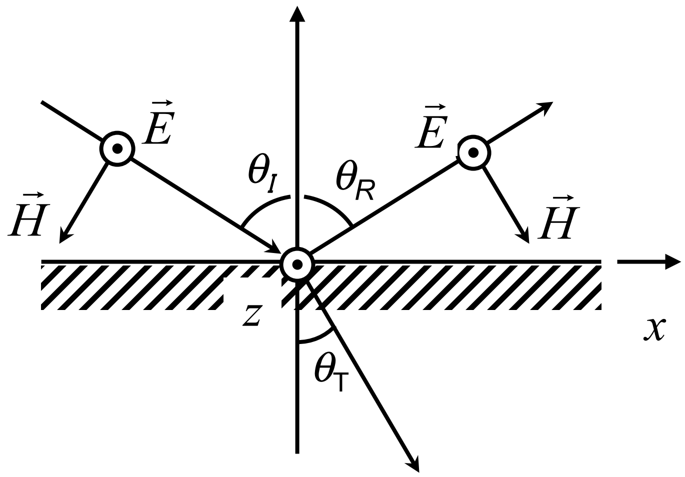
radial rays spring from the transmitting antenna

- when a ray impinges on the <u>plane surface</u> the corresponding wave is reflected and transmitted, thus generating **reflected** and **transmitted** rays

- The incident ray trajectory is modified according to the *Snell's laws of reflection (transmission)*. Rays and wavefronts are *as if* the reflected wave generated at the source image point…

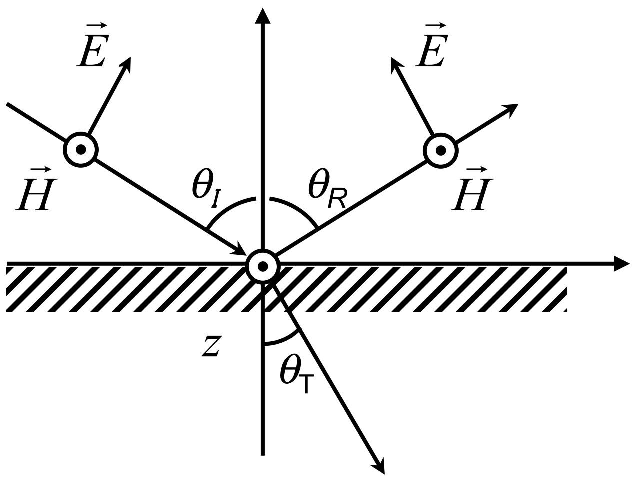
- The field amplitude / phase change at the interaction point according to proper Fresnel's *reflection (transmission) coefficients*

Reflection and Transmission Coefficients



• TE polarization

$$\Gamma_{TE} = \frac{\cos\theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}} \; ; \; \tau_{TE} = 1 + \Gamma_{TE}$$

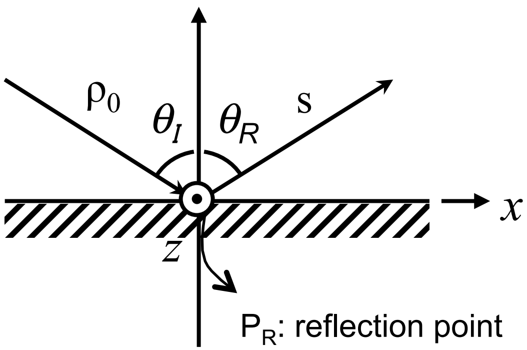


• TM polarization

$$\Gamma_{TM} = \frac{\left(\frac{n_2}{n_1}\right)^2 \cos\theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}{\left(\frac{n_2}{n_1}\right)^2 \cos\theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}} \; ; \; \tau_{TM} = 1 + \Gamma_{TM}$$

Field formulation

Reflected ray

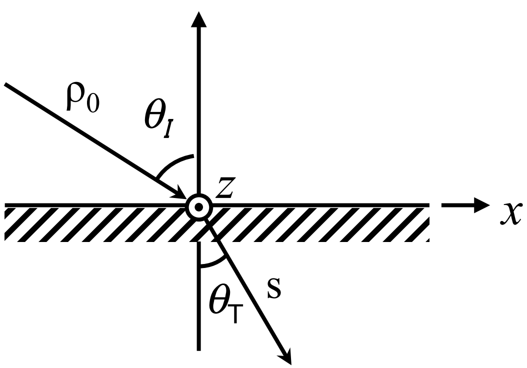


- trajectory: reflection law or Fermat's principle
- Field expression:

$$\begin{bmatrix} \vec{E}_r^{TE}(s) \\ \vec{E}_r^{TM}(s) \end{bmatrix} = \begin{bmatrix} \Gamma_{TE} & 0 \\ 0 & \Gamma_{TM} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_i^{TE}(P_R) \\ \vec{E}_i^{TM}(P_R) \end{bmatrix} \cdot \frac{\rho_0}{\rho_0 + s} e^{-j\beta s}$$

Spreading factor

Transmitted ray



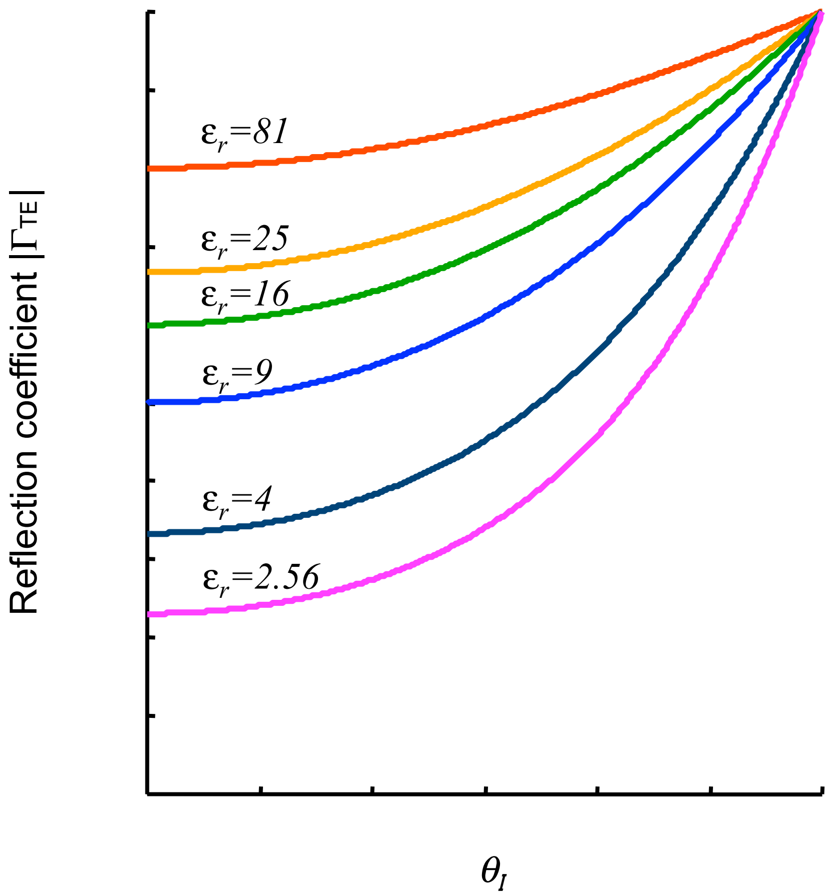
- direction: Snell's law or Fermat's principle
- Field expression:

$$\begin{bmatrix} \vec{E}_t^{TE}(s) \\ \vec{E}_t^{TM}(s) \end{bmatrix} = \begin{bmatrix} \tau_{TE} & 0 \\ 0 & \tau_{TM} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_i^{TE}(P_R) \\ \vec{E}_i^{TM}(P_R) \end{bmatrix} \cdot \frac{\rho_0}{\rho_0 + s} e^{-j\beta' s}$$

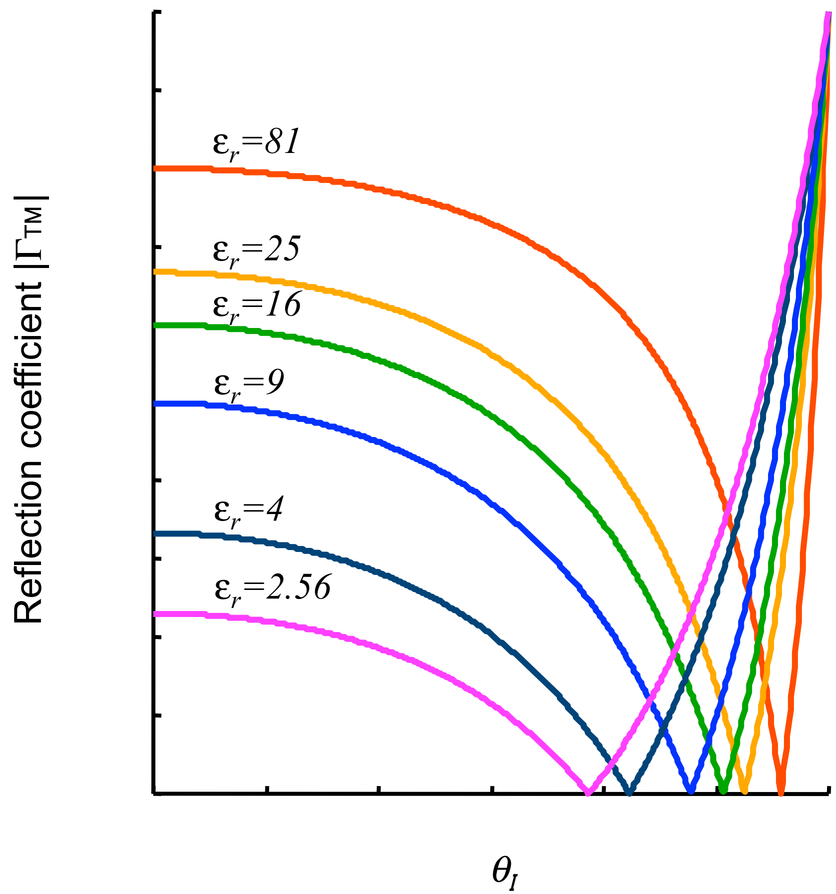
Fresnel's coefficients

- <u>Reflection does not change the spreading factor of the wave !!</u>

Example: dielectric materials



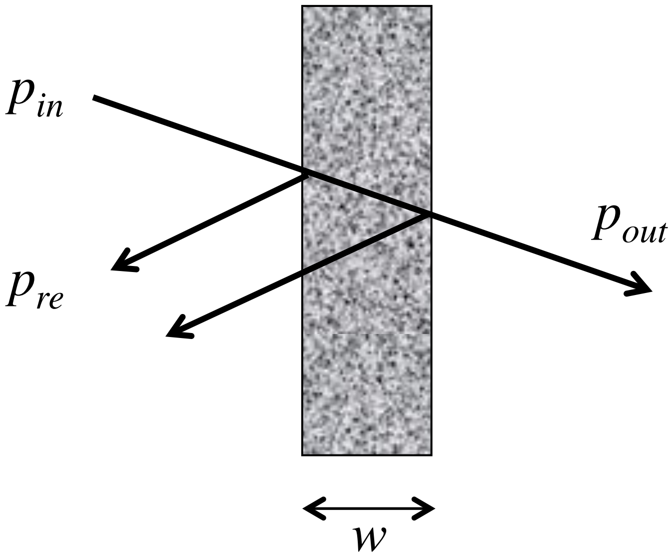
TE Polarization



TM Polarization

Transmission through a wall (1/5)

* Hypotheses: - normal or quasi-normal incidence
  - weakly lossy medium



$$\Gamma \approx \Gamma_{TE} = \frac{\cos\theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}} = \frac{1-\sqrt{\varepsilon_r}}{1+\sqrt{\varepsilon_r}}$$

$$\frac{S_{re}}{S_{in}} = \frac{|E_{re}|^2 / 2\eta}{|E_{in}|^2 / 2\eta} = \frac{|E_{re}|^2}{|E_{in}|^2} \approx |\Gamma|^2$$

(Source: Prof. H.L. Bertoni)

Transmission through a wall (2/5)

In a lossy medium the wavenumber can be written as:

$$k = \omega\sqrt{\mu_0\varepsilon_c} = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r}$$

The complex relative dielectric constant can be written as:

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' = \frac{\varepsilon}{\varepsilon_0} - j\frac{\sigma}{\omega\varepsilon_0}$$

If the medium is <u>weakly lossy</u> ε” << ε’ .
A plane wave propagating through the lossy medium has the expression:

$$\mathbf{E} = \mathbf{E_0}e^{-jkr} = \mathbf{E_0}e^{-(\alpha+j\beta)r};\ \text{with jk}=\alpha+\text{j}\beta$$

Thus:

$$k = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_r} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r' - j\varepsilon_r''} = \frac{\omega}{c}\sqrt{\varepsilon_r' - j\varepsilon_r''} =$$

$$= \frac{\omega}{c}\sqrt{\varepsilon_r'}\sqrt{1 - j\frac{\varepsilon_r''}{\varepsilon_r'}} \approx \frac{\omega}{c}\sqrt{\varepsilon_r'}\left(1 - j\frac{\varepsilon_r''}{2\varepsilon_r'}\right)$$

Where the series expansion have been truncated at first order

Transmission through a wall (3/5)

Therefore:

$$jk = \alpha + j\beta \approx \frac{\omega}{c}\sqrt{\varepsilon_r'}\left(\frac{\varepsilon_r''}{2\varepsilon_r'} + j\right) \Rightarrow$$

$$\begin{cases} \alpha \approx \dfrac{\omega}{c}\sqrt{\varepsilon_r'}\left(\dfrac{\varepsilon_r''}{2\varepsilon_r'}\right) \\ \beta \approx \dfrac{\omega}{c}\sqrt{\varepsilon_r'} \end{cases}$$

$$\left|\overline{E}(r)\right| = \left|\overline{E}(0)\right| \cdot e^{-\alpha r}$$

$$S(r) = S(0) \cdot e^{-2\alpha r}$$

Transmission through a wall (4/5)

The reflection coefficient at normal incidence for the air-medium interface is

$$\Gamma_{0m} = \frac{1-\sqrt{\varepsilon_r}}{1+\sqrt{\varepsilon_r}}$$

The reflection coefficient for the second, medium-air interface is (see the expression of the reflection coefficients for normal incidence)

$$\Gamma_{m0} = \frac{\sqrt{\varepsilon_r}-1}{1+\sqrt{\varepsilon_r}} = -\Gamma_{0m}$$

Now if we consider the first interface we have

$$\frac{S_{refl1}}{S_{inc1}} = \frac{\left|\vec{E}_{refl1}\right|^2}{\left|\vec{E}_{in1}\right|^2} = \left|\Gamma_{0m}\right|^2$$

Transmission through a wall (5/5)

For power conservation we have:

$$S_{inc1} = S_{refl1} + S_{trasm1} \quad \Rightarrow \quad 1 = \frac{S_{refl1}}{S_{inc1}} + \frac{S_{trasm1}}{S_{inc1}} = \left|\Gamma_{0m}\right|^2 + \frac{S_{trasm1}}{S_{inc1}}$$

$$\frac{S_{trasm1}}{S_{inc1}} = 1 - \left|\Gamma_{0m}\right|^2$$

Now the transmitted power at the first interface, properly multiplied by the lossy-medium attenuation factor becomes the incident power at the second interface, therefore we have

$$\frac{S_{refl2}}{S_{inc2}} = \left|\Gamma_{m0}\right|^2 = \left|\Gamma_{0m}\right|^2 = \left|\Gamma\right|^2 \quad ; \quad \frac{S_{transm2}}{S_{inc2}} = \frac{S_{transm2}}{S_{transm1} e^{-2\alpha w}} = \frac{S_{transm2}}{S_{inc1}\left(1 - \left|\Gamma\right|^2\right) e^{-2\alpha w}} = 1 - \left|\Gamma\right|^2$$

Thus:

$$\frac{S_{transm2}}{S_{inc1}} = \frac{S_{out}}{S_{in}} = \left(1 - \left|\Gamma\right|^2\right)^2 e^{-2\alpha w} \Rightarrow L_t = \frac{S_{in}}{S_{out}} = \frac{e^{2\alpha w}}{\left(1 - \left|\Gamma\right|^2\right)^2}$$

Example of Transmission Loss

Brick wall: $\varepsilon_r'=4$, $\varepsilon_r''=0.2$, $w=20$ cm

$$\left|\Gamma\right|^2 = \frac{S_{refl1}}{S_{inc1}} \approx \left|\frac{\sqrt{4}-1}{\sqrt{4}+1}\right|^2 = \frac{1}{9} = 0.11 \text{ or } -9.6\text{dB}$$

at 1800 MHz (λ_o=1/6 m): $\alpha = \dfrac{0.2\pi}{(1/6)\sqrt{4}} = 1.88$

$$\mathrm{L}_t = \frac{S_{in}}{S_{out}} = (1-0.11)^2 e^{2(0.2)(1.88)} = 2.7 \text{ or } 4.3\text{dB}$$

Summary of Reflection and Transmission Loss

Theory

Wall Type	Frequency Band	Ref. loss	Trans. Loss
Brick, exterior	1.8 - 4 GHz	10 dB	10 dB
Concrete block, interior	2.4 GHz		5 dB
Gypsum board, interior	3.4 GHz	4 dB	2 dB

Measured

Exterior frame with metal siding	800 MHz 5 - 6 GHz 5 GHz		4 - 7 dB 9 - 18 dB 36 dB
Brick, exterior	4 - 6 GHz	10 dB	14 dB
Concrete block, interior	2.4 / 5 GHz		5 / 5 - 10 dB
Gypsum board, interior	2.4 / 5 GHz		3 / 5 dB
Wooden floors	5 GHz		9 dB
Concrete floors	900 MHz		13 dB

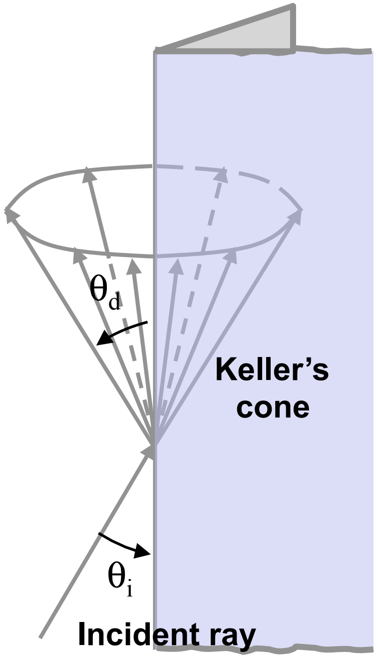
(Source: Prof. H.L. Bertoni)

Geometrical Theory of Diffraction

The extension of GO to the category of diffracted rays was first introduced by J. B. Keller in 1961 and is based on the following assumptions[6] :

I. *A diffracted ray is generated whenever a ray impinges on an edge (or on a vertex)*

II. *For every diffracted ray the Fermat's principle holds*

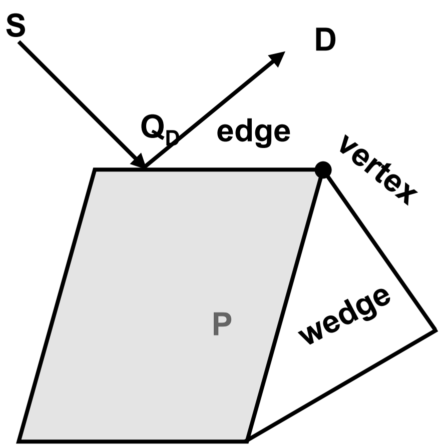


***Diffraction law*: the angles between incident / diffracted ray and the edge satisfy "Snell's law applied to diffraction":**

$$n_i \cdot \sin\theta_i = n_d \cdot \sin\theta_d$$

➔ If the rays are in the same material then: $\theta_d=\theta_i$;
Therefore diffracted rays ouside the wedge belong to the ***Keller's cone***

The diffracted ray (1/3)



- In urban propagation only straight edges (local field principle) are of interest. Vertex diffraction won't be treated here

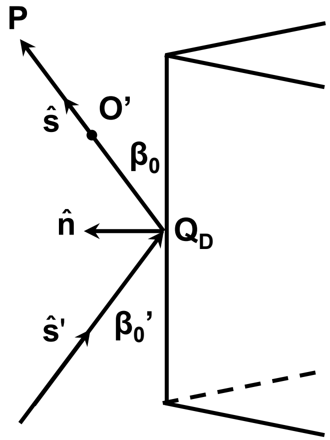
- If the impinging wave is plane (or can be approximated so for the local field principle) then the diffracted wave is cylindrical for perpendicular incidence ($\theta_d=\theta_i=\pi/2$) and conical for oblique incidence (the wavefront is a cone) [7]

- The diffracted wave is so that one caustic coincides with the edge. **Therefore the divergence factor of the diffracted wave/ray is different from that of the incident wave/ray** (see further on)

- The diffracted ray field can be computed by solving Maxwell's equations for a plane , cylindrical or spherical wave incident on a straight conducting edge [7, 8, 9] and somehow subtracting from the solution the incident wave and the reflected wave(s).

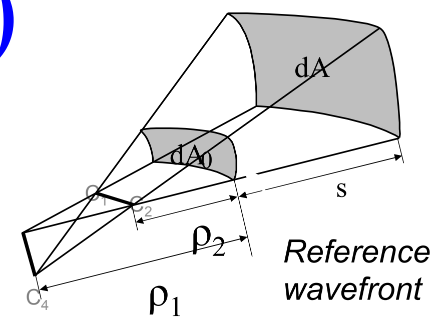
- Then the diffracted field is expanded in a Luneberg-Kline series from which only the first term (high frequency approx.) is kept in order to derive the ***diffraction coefficients***

The diffracted ray (2/3)



The high frequency term has the form:

$$\vec{E}^d(s) = \vec{E}^d(O') \cdot \sqrt{\frac{\rho_1^d \cdot \rho_2^d}{(\rho_1^d + s)\cdot(\rho_2^d + s)}} \cdot e^{-j\beta s}$$



ρ_1^d, ρ_2^d = curvature radii of the diffracted wave.

One caustic coincides with the edge: ρ_2^d corresponds to O'-Q_D where O' is the reference point, origin of the coordinate s.

It is useful to choose O'=Q_D ($\rho_2^d=0$ ➔ simpler expression). However for power conservation reasons $\mathbf{E}^d(O') \to \infty$ for $O' \to Q_D$

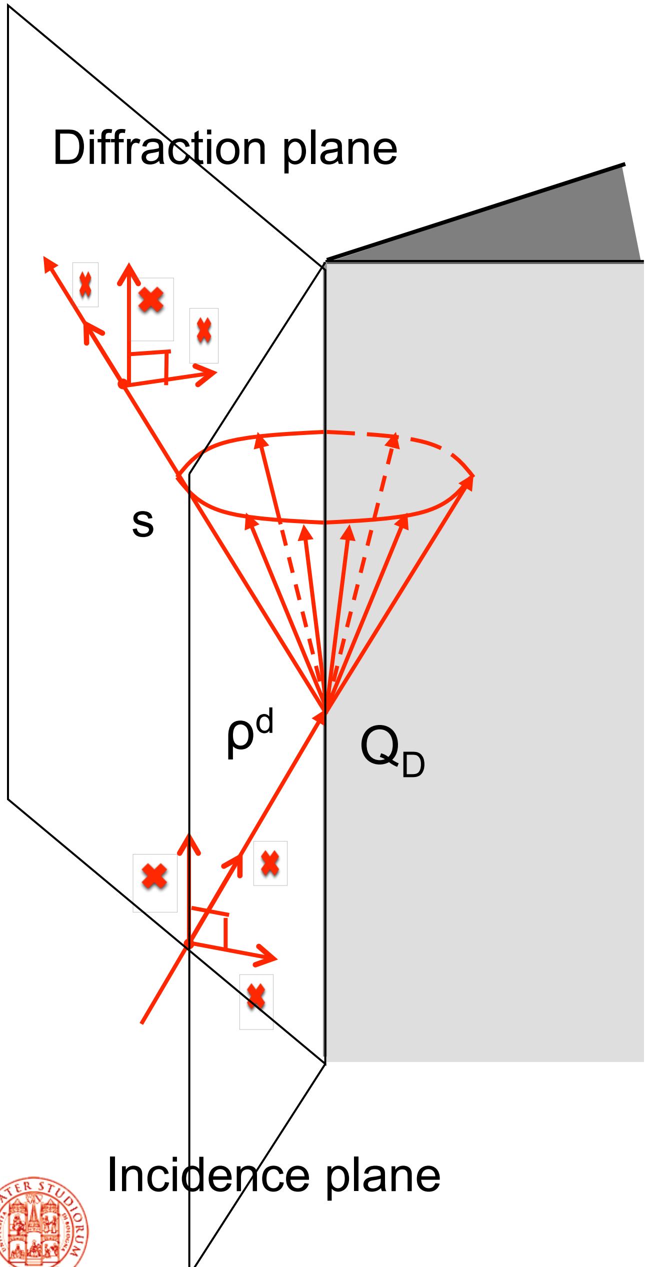
Since $\mathbf{E}^d(s)$ cannot change with the reference system, therefore it must be:

$$\lim_{\substack{O' \to Q_D \\ (\rho_2^d \to 0)}} \left[\vec{E}^d(O') \cdot \sqrt{\rho_2^d} \right] = \textit{finite vector} \equiv \vec{E}^i(Q_D) \cdot \mathbf{D} \quad \Longrightarrow \quad \boxed{\vec{E}^d(s) = \vec{E}^i(Q_D) \cdot \mathbf{D} \cdot A(\rho^d, s) \cdot e^{-j\beta s}}$$

with: $A(\rho^d, s) = \sqrt{\dfrac{\rho^d}{(\rho^d + s)\cdot s}}$

D is the ***diffraction matrix,*** which contains the diffraction coefficients

The diffracted ray (3/3)



- ➔ trajectory: Fermat's principle
- ➔ Field expression:

$$\begin{bmatrix} E^d_{\beta_0} \\ E^d_{\phi} \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & D_h \end{bmatrix} \begin{bmatrix} E^i_{\beta_0'}(Q_D) \\ E^i_{\phi'}(Q_D) \end{bmatrix} \cdot A(s,\rho^d) \cdot e^{-j\beta s}$$

spreading factor…

if the proper local reference system is adopted (see figure) then the diffraction matrix reduces to a 2x2 diagonal matrix, otherwise it's a 3x3 matrix

Φ-polarization is called "hard" (TE), β-polarizationi is called "soft" (TM)

The divergence factor

If $\rho_2^{d} \to 0$ as shown, then we get :
$(\rho_1^{d} \to \rho^{d})$

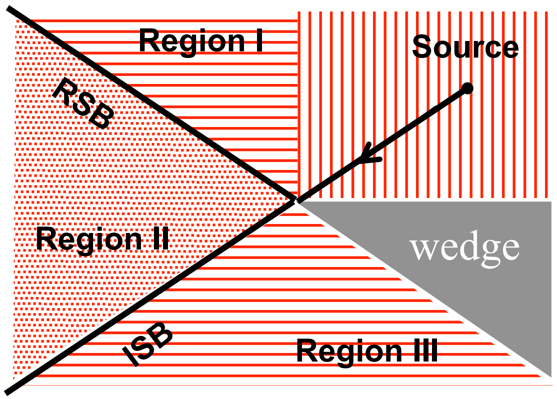
$$A\left(\rho^{d}, s\right) = \sqrt{\frac{\rho^{d}}{s \cdot \left(\rho^{d} + s\right)}}$$

For a straight edge we have:

$$A\left(\rho^{d}, s\right) = \begin{cases} \dfrac{1}{\sqrt{s}} & \text{for a plane incident wave} \\ \dfrac{1}{\sqrt{s \cdot \sin \beta_o^{'}}} & \text{for a cylindrical incident wave} \\ \sqrt{\dfrac{\rho^{d}}{s \cdot \left(\rho^{d} + s\right)}} & \text{for a spherical incident wave} \end{cases}$$

- For the computation of the diffraction coefficients we refer in the following to a simple case with a cylindrical incident wave.

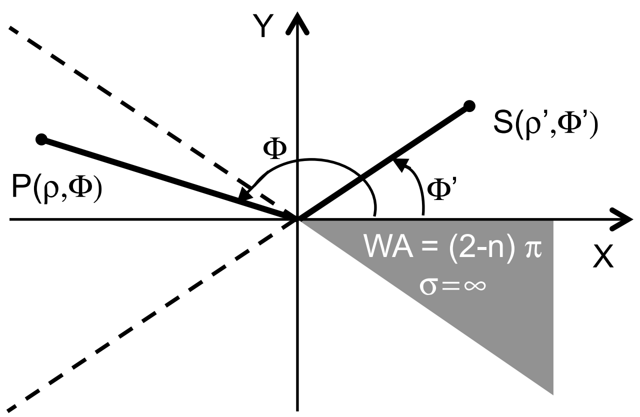
The diffraction coefficients for a canonical 2D problem



ISB : Incidence Shadow Boundary

RSB : Reflection Shadow Boundary

R I : direct + reflected + diffracted
R II : direct + diffracted
R III : diffracted



Hypotheses:

- unlimited perfectly conducting wedge of angular width $WA = (2-n)\pi \quad (0 \le n < 2)$
- Infinite uniform linear source parallel to the edge with constant current $I_0\, \mathbf{i}_z$

⇩

cylindrical incident wave with normal incidence

The diffraction coefficients

Adopting the method described above the following Keller's diffraction coefficients are obtained (***G****eometrical* ***T****heory of* ***D****iffraction, GTD*) [9]

$$D^S(\phi,\phi',n) = \frac{-e^{-j\pi/4}\cdot\sin\left(\pi/n\right)}{n\sqrt{2\pi\beta}}\cdot\left[\frac{1}{\cos\left(\pi/n\right)-\cos\left(\xi^-/n\right)} - \frac{1}{\cos\left(\pi/n\right)-\cos\left(\xi^+/n\right)}\right]$$

$$\xi^- = \Phi - \Phi'$$
$$\xi^+ = \Phi + \Phi'$$

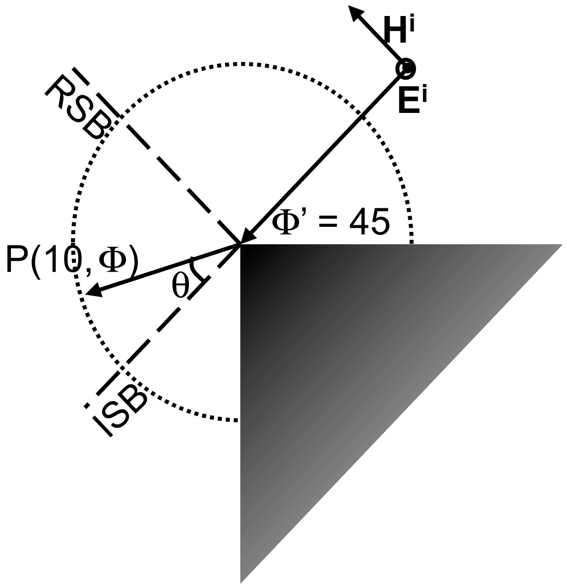
$$D^H(\phi,\phi',n) = \frac{-e^{-j\pi/4}\cdot\sin\left(\pi/n\right)}{n\sqrt{2\pi\beta}}\cdot\left[\frac{1}{\cos\left(\pi/n\right)-\cos\left(\xi^-/n\right)} + \frac{1}{\cos\left(\pi/n\right)-\cos\left(\xi^+/n\right)}\right]$$

Such coefficients have singularities on the shadow boundaries, i.e. when:

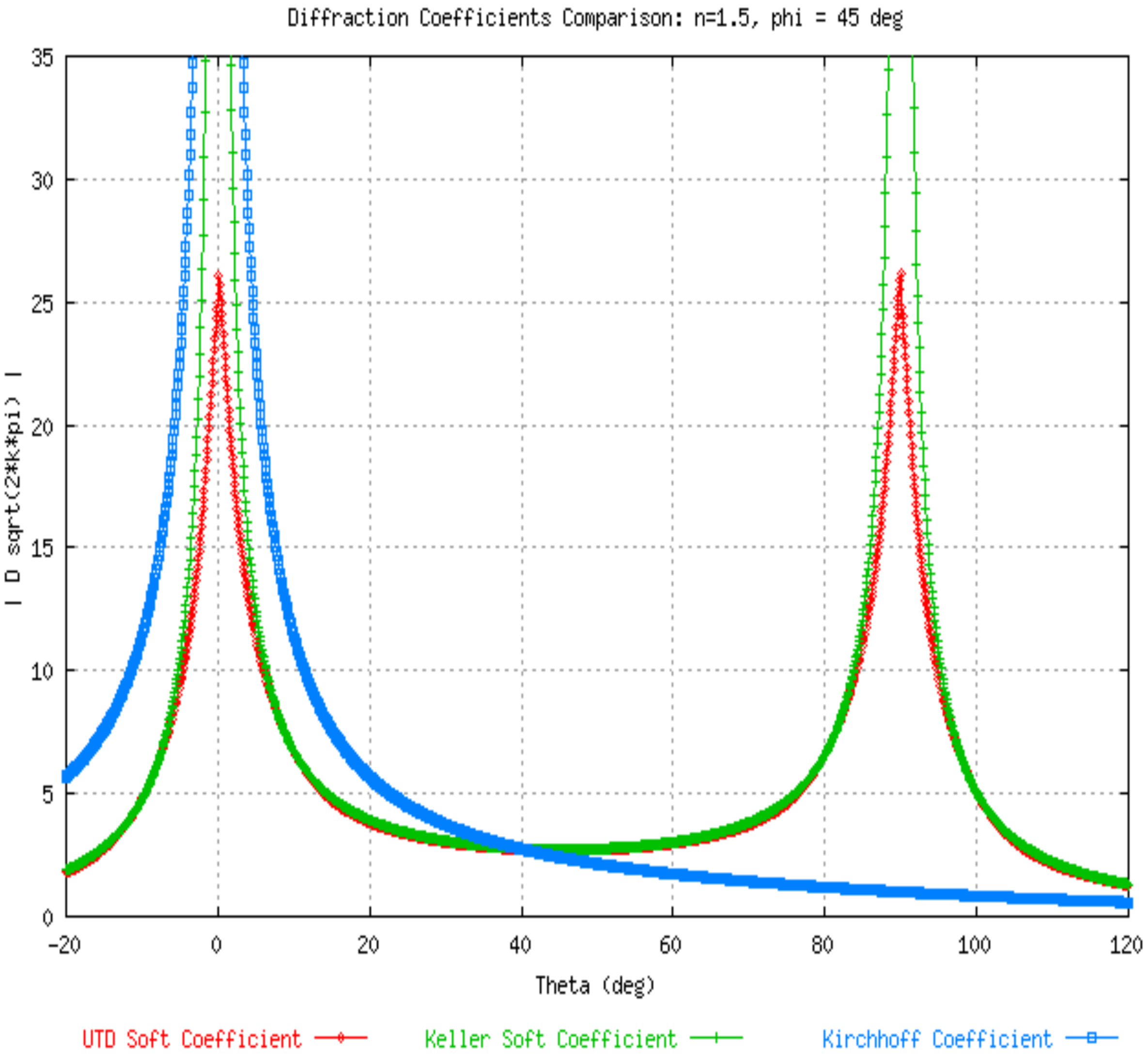
$$\xi^- = \phi - \phi' = \pi \quad \text{(ISB)}$$
$$\xi^+ = \phi + \phi' = \pi \quad \text{(RSB)}$$

Therefore also other, more complicated coefficients have been derived which do not have such singularity: the UTD (***U****niform* ***T****heory of* ***D****iffraction*) coefficients

Example (1/2)



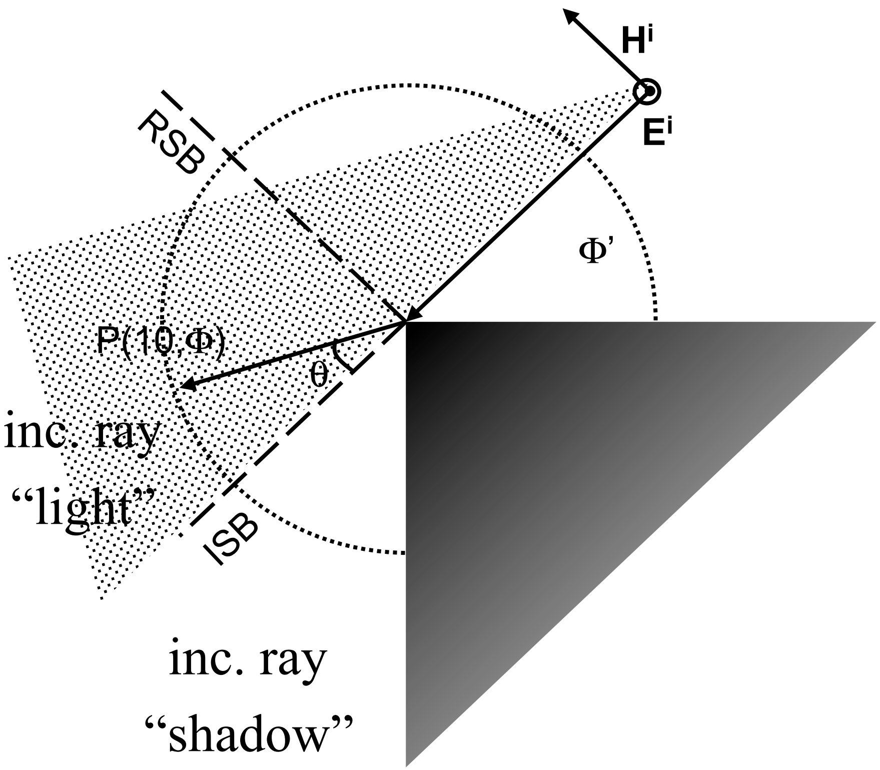
H^i
E^i
RSB
ISB
Φ' = 45
P(10, Φ)
θ

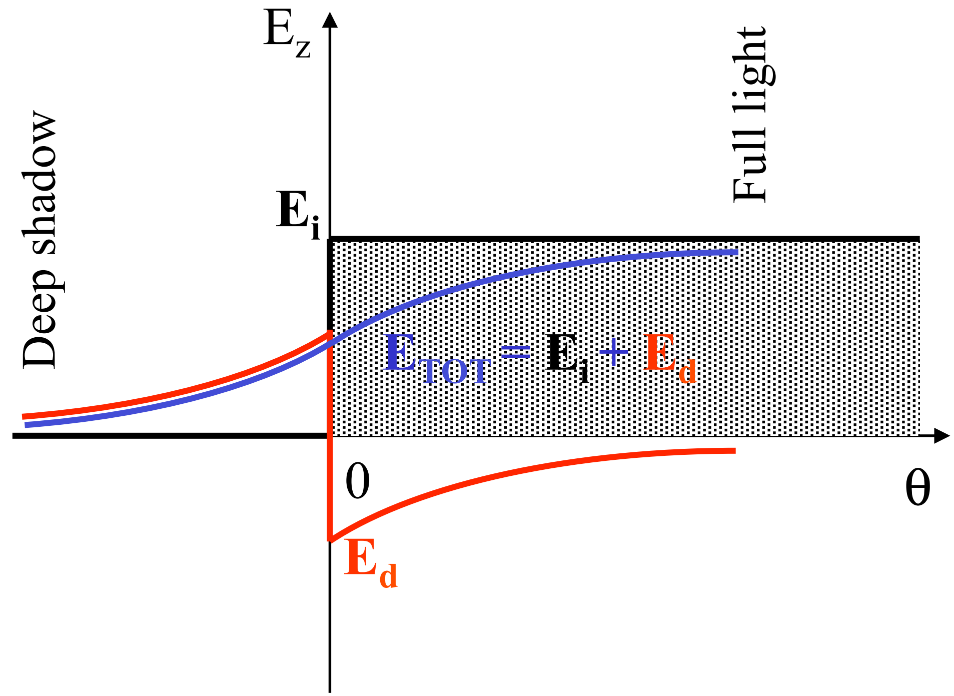


Diffraction Coefficients Comparison: n=1.5, phi = 45 deg
| D sqrt(2*k*pi) |
Theta (deg)
UTD Soft Coefficient
Keller Soft Coefficient
Kirchhoff Coefficient

Example (2/2)

UTD, considering the diffracted ray and the incident ray





Other notes on GTP

- A single ray can undergo multiple interactions. The resulting ray is therefore a polygonal line and the proper interaction coefficients must be applied for each interaction. The proper divergence factor must then be applied for the overall piece-wise path.

- Reflection and transmission does not change the form of the divergence factor of a ray. Diffraction does.

- Diffraction coefficients for oblique incident and dielectric wedges have also been derived by some authors

- The interaction called "diffuse scattering" is important but is not treated here. It will be briefly treated further on.

Computation Examples: reflection

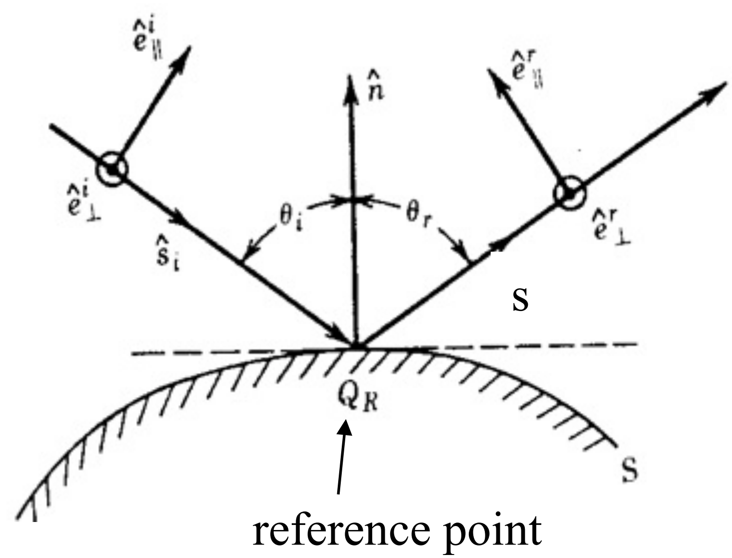
For the generic incident astigmatic wave we can write:

$$\vec{E}_r(s) = \underbrace{\vec{E}(Q_R)}_{\text{field at reference point } (Q_R,\ s=0)} \cdot \underbrace{\underline{\underline{\mathbf{R}}}(Q_R, \theta_i)}_{\text{Reflection coefficient (Dyadic)}} \cdot \underbrace{\sqrt{\frac{\rho_1 \cdot \rho_2}{(\rho_1 + s)\cdot(\rho_2 + s)}}}_{\text{divergence or spreading factor}} \cdot \underbrace{e^{-j\beta s}}_{\text{Phase factor}}$$

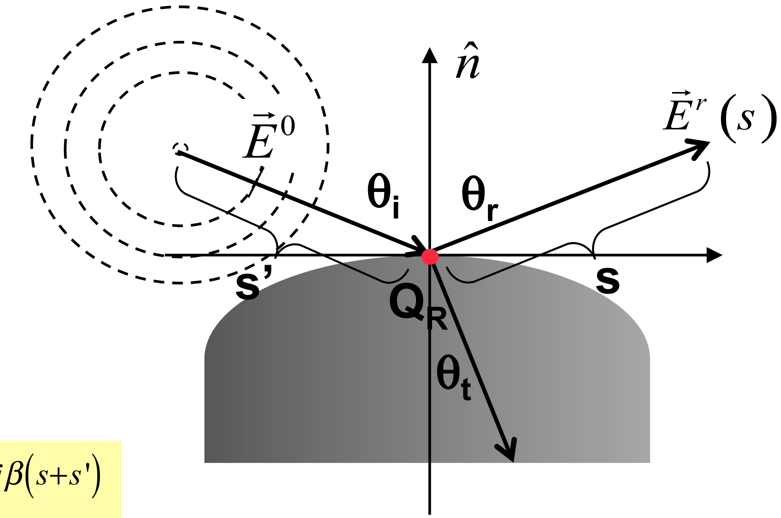
The use of the Dyadic Reflection coefficient [8] allows to refer to a fixed reference system

$$\underline{\underline{\mathbf{R}}} = \Gamma_{//}\left(\hat{e}^i_{//}\hat{e}^r_{//}\right) + \Gamma_{\perp}\left(\hat{e}^i_{\perp}\hat{e}^r_{\perp}\right)$$

$$\left(\bar{a}\,\bar{b}\right) \triangleq \begin{pmatrix} a_x b_x & a_x b_y & a_x b_z \\ a_y b_x & a_y b_y & a_y b_z \\ a_z b_x & a_z b_y & a_z b_z \end{pmatrix}$$



Reflection (II)



For a spherical incident wave the expression above becomes ($\rho_1 = \rho_2 = s'$):

$$\vec{E}_r(s) = \vec{E}^0 \frac{e^{-j\beta s'}}{s'} \cdot \underline{\underline{\mathbf{R}}} \cdot \frac{s'}{s+s'} e^{-j\beta s} = \vec{E}^0 \cdot \underline{\underline{\mathbf{R}}} \cdot \frac{e^{-j\beta(s+s')}}{s+s'}$$

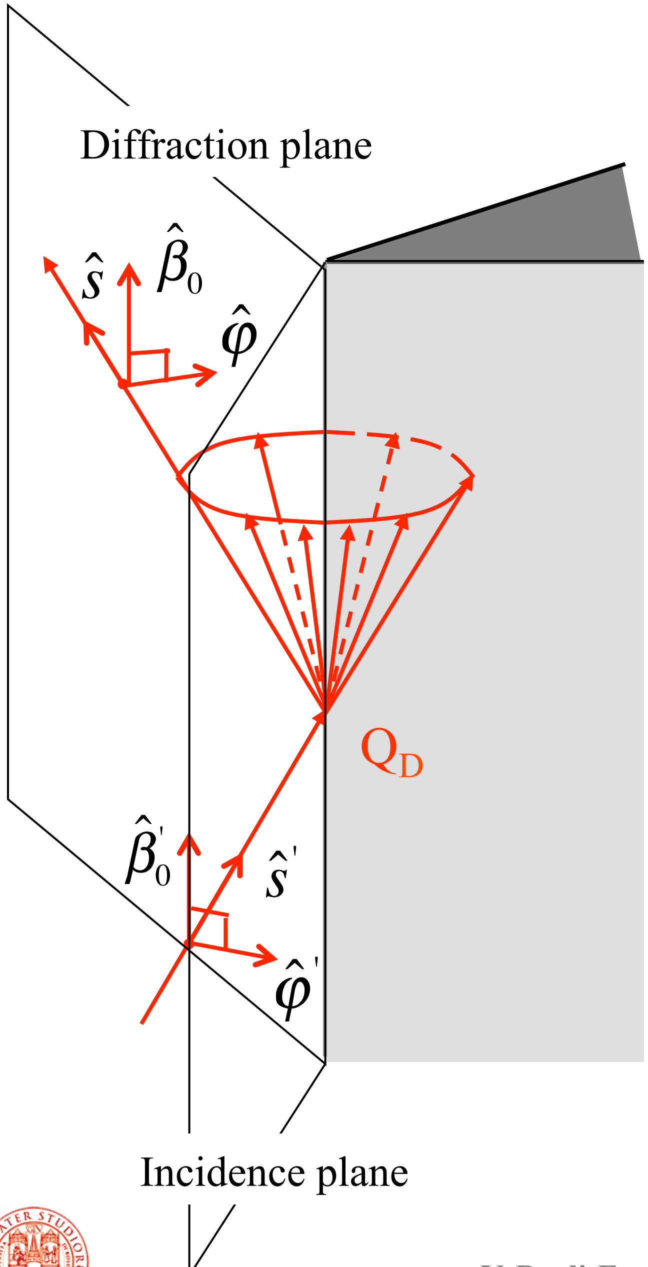
which is equivalent to

$$\begin{bmatrix} \vec{E}_{r\ TE}(s) \\ \vec{E}_{r\ TM}(s) \end{bmatrix} = \begin{bmatrix} \Gamma_{TE} & 0 \\ 0 & \Gamma_{TM} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}^0_{TE} \\ \vec{E}^0_{TM} \end{bmatrix} \cdot \frac{e^{-j\beta s'}}{s'} \frac{s'}{s+s'} e^{-j\beta s}$$

Divergence factor for a spherical wave

Incident field in Q_R

Diffraction



Diffraction coefficients ➔ Diffracted field

$$\begin{bmatrix} E^d_{\beta_0} \\ E^d_{\phi} \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & D_h \end{bmatrix} \begin{bmatrix} E^i_{\beta_0^{'}}(Q_D) \\ E^i_{\phi'}(Q_D) \end{bmatrix} \cdot A \cdot e^{-j\beta s}$$

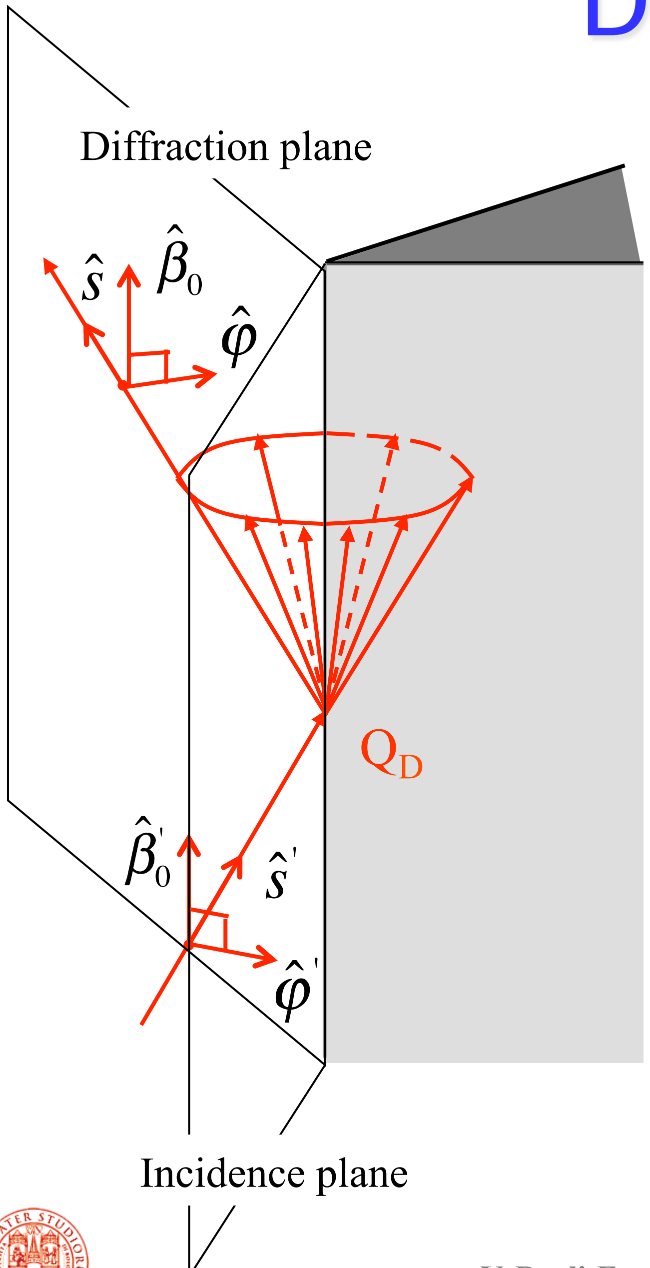
A is the *divergence factor* for the diffracted field. For a spherical incident wave:

$$A(s',s) = \sqrt{\frac{s'}{s \cdot (s'+s)}} \qquad \vec{E}^i(Q_D) = \vec{E}^{0i} \frac{e^{-j\beta s'}}{s'}$$

Therefore we have:

$$\begin{bmatrix} \vec{E}^d_{\beta_0} \\ \vec{E}^d_{\phi} \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & D_h \end{bmatrix} \begin{bmatrix} \vec{E}^{0i}_{\beta_0^{'}} \\ \vec{E}^{0i}_{\phi'} \end{bmatrix} \cdot \frac{1}{\sqrt{s \cdot s' \cdot (s'+s)}} \cdot e^{-j\beta(s+s')}$$

Diffraction (II)



Using the the Dyadic Diffraction coefficient:

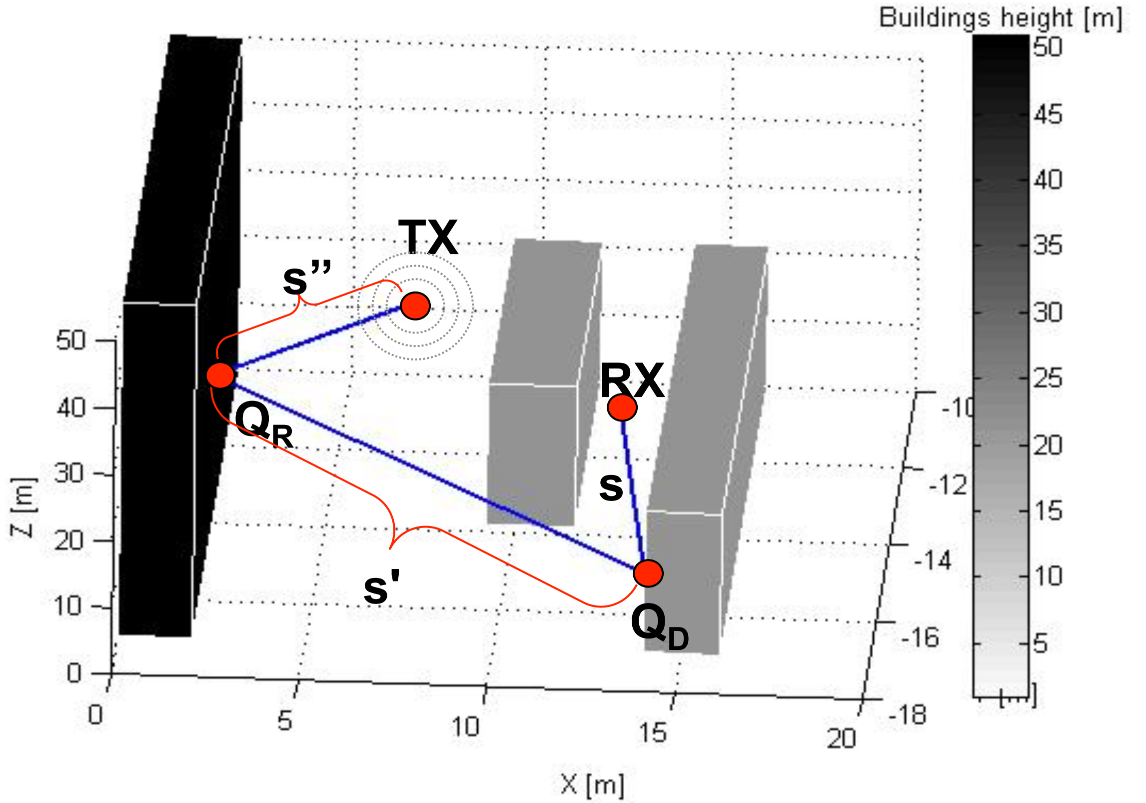
$$\underline{\underline{\mathbf{D}}} = D_s \left(\hat{\beta}_0' \hat{\beta}_0\right) + D_h \left(\hat{\phi}' \hat{\phi}\right)$$

we have

$$\bar{E}^d = \bar{E}^0 \cdot \underline{\underline{\mathbf{D}}} \cdot \frac{1}{\sqrt{s \cdot s' \cdot \left(s' + s\right)}} \cdot e^{-j\beta\left(s+s'\right)}$$

Double interaction (1/2)

Reflection + Vertical Edge Diffraction



Field at the reflection point: $\vec{E}(Q_R) = \vec{E}^0 \dfrac{e^{-j\beta s''}}{s''}$

Double interaction (2/2)

The field at the diffraction point is:

$$\vec{E}(Q_D) = \underbrace{\vec{E}_0 \cdot \frac{e^{-j\beta s''}}{s''}}_{\vec{E}(Q_R)} \cdot \underline{\underline{\mathbf{R}}} \cdot \frac{s''}{s'+s''} e^{-j\beta s'} = \vec{E}_0 \cdot \underline{\underline{\mathbf{R}}} \cdot \frac{e^{-j\beta(s'+s'')}}{s'+s''}$$

Finally, the field at the RX can be computed as:

$$\vec{E}(Rx) = \vec{E}(Q_D) \cdot \underline{\underline{\mathbf{D}}} \cdot \sqrt{\frac{(s'+s'')}{s\left[s+(s'+s'')\right]}} \cdot e^{-j\beta s} =$$

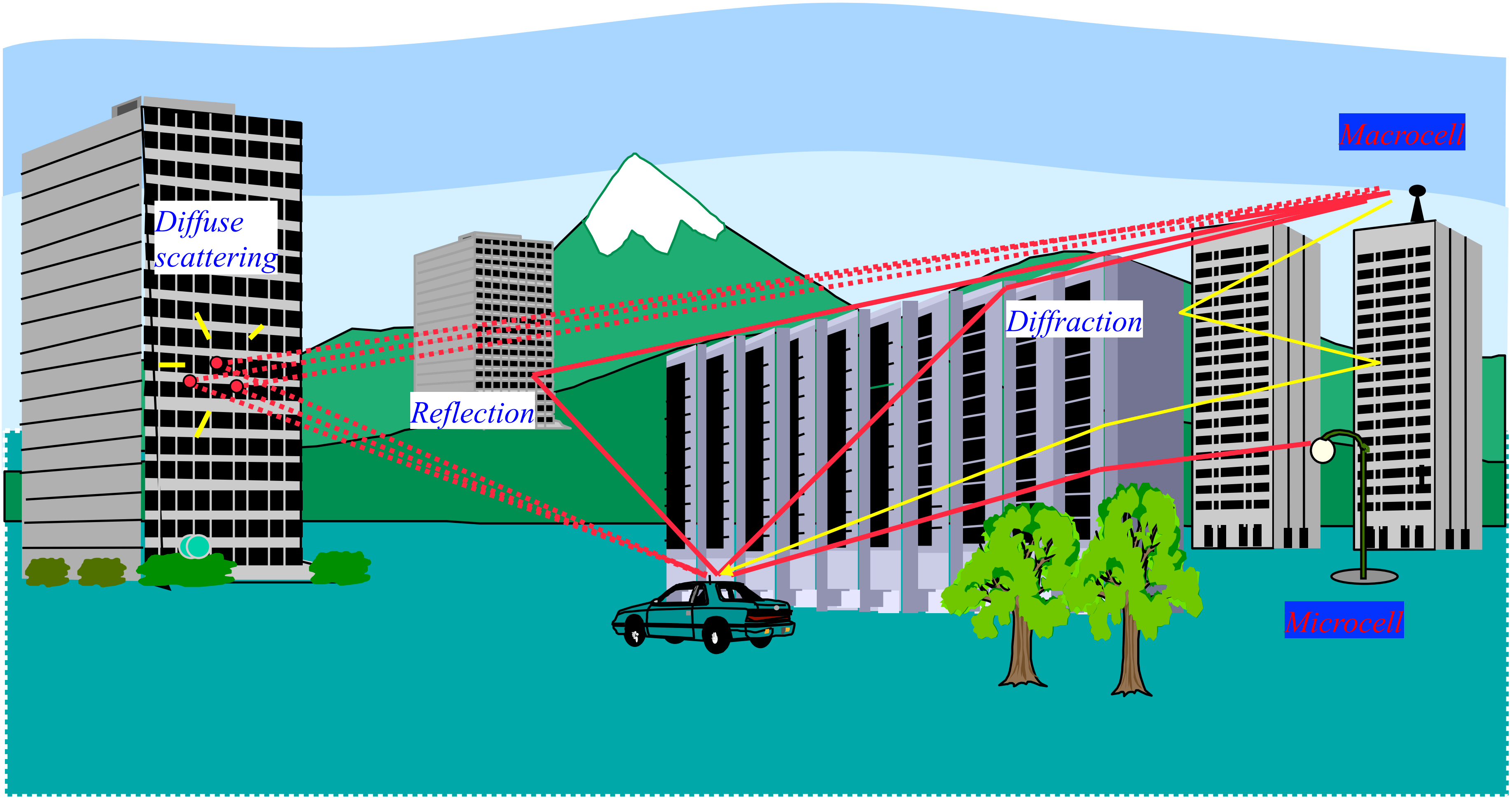
$$= \vec{E}^0 \cdot \underline{\underline{\mathbf{R}}} \cdot \underline{\underline{\mathbf{D}}} \cdot \frac{1}{s'+s''} \cdot \sqrt{\frac{(s'+s'')}{s\left[s+(s'+s'')\right]}} \cdot e^{-j\beta(s+s'+s'')} =$$

$$= \vec{E}^0 \cdot \underline{\underline{\mathbf{R}}} \cdot \underline{\underline{\mathbf{D}}} \cdot \frac{1}{\sqrt{s(s'+s'')(s+s'+s'')}} \cdot e^{-j\beta(s+s'+s'')}$$

$$\left(\begin{array}{l} remember: \\ A(s',s) = \sqrt{\dfrac{s'}{s\cdot(s'+s)}} \end{array} \right)$$

Superposition of multiple rays (1/3)

(Multipath propagation…)



Superposition of multiple rays (2/3)

The total field at a given position P can be computed through a coherent, vectorial sum of the field of all rays reaching P (difficult to determine though…):

$$\bar{E}(P) = \sum_{k=1}^{N_r} \bar{E}^k(P)$$

Moreover, the delays and angles of departure/arrival of the different ray contributions can be recorded get a multidimensional prediction.
In fact the GTP, determining its trajectory, also yields the following parameters for the k-th ray:

s^k total unfolded length

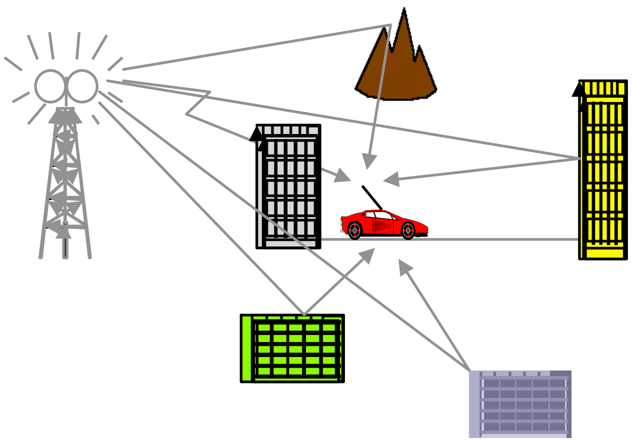
$t^k = s^k / c$ propagation delay

$\chi^k \equiv (\theta_T^k, \phi_T^k)$ angles of departure

$\psi^k \equiv (\theta_R^k, \phi_R^k)$ angles of arrival

Superposition of multiple rays (3/3)

Multipath propagation → not only attenuation !



Multipath

- Attenuation (fading)
- Time dispersion — *Delay spread*
- Angle/space dispersion — *Angle spread*

Some systems can *exploit* multipath, others only *cope with* it

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