

C – IL CANALE RADIOMOBILE

- **Caratterizzazione deterministica del canale radiomobile**
 - **Fuzioni di trasferimento del canale: caso statico e dinamico**
 - **Fading piatto e selettivo**
 - **I parametri sintetici (delay-spread, banda di coerenza ecc.).**
 - **Estensione al dominio spaziale**
 - **Attocorrelazioni**
 - **Esempi**
- **Tecniche di diversità, MIMO, e space-time coding**
 - **Tecniche di diversità**
 - **Matrice di canale e MIMO**
 - **Multiplexing gain e cenni a space-time coding.**

Coping with fading-affected channels

- Mobile radio channels generally show (flat or selective) multipath fading in frequency, time and space, i.e. in “e-kind” domains → fading and/or distortion of the received signal → channel capacity limitation, where channel capacity is defined as the maximum bitrate with a virtually zero **Bit-Error-Rate (BER)** or **Symbol-Error-Rate (SER)**
- When fading is flat then its effects (high BER) can be *reduced* with an increase in Tx power, although this is not always the best policy (e.g: for interference problems)
- When fading is selective then there is often **irreducible BER** due to signal distortion
- In modern digital systems the distinction btw flat and selective fading is not sharp anymore. Anyway fading yields quality degradation, i.e. BER increase
- In order to decrease the effect of multipath fading proper techniques can be adopted:
 - **Coding**
 - **Diversity**
 - **Equalization**
 - ...



Diversity (1/5)

- Diversity techniques can combat flat fading. Diversity involves providing replicas of the transmitted signal over time, frequency, or space.
- The number of replicas (*branches*) is related to the diversity order
- Thanks to the low channel correlation after a given (time-, frequency-, space-) distance by combining these replicas a diversity gain in terms of Signal to Noise Ratio (SNR) or BER is obtained.

Example: $z=x+y$ with x and y independent Gaussian random signal envelopes. The standard dev. and the mean of z are: $\sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2}$; $\mu_z = \mu_x + \mu_y$

Therefore z is “less variable” because $\sigma_z / \mu_z < \sigma_{x,y} / \mu_{x,y} \rightarrow$ lower fading margin

- There are three main types of diversity schemes in wireless communications, enforced in *e-kind* domains:



TIME DIVERSITY

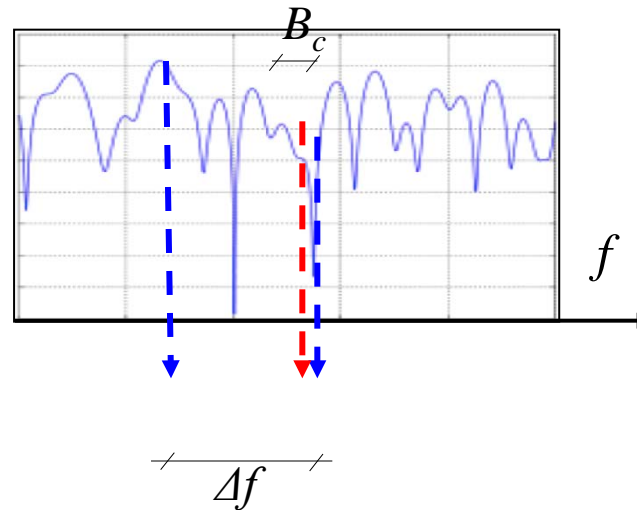
FREQUENCY DIVERSITY

SPACE DIVERSITY



Diversity (2/5)

- **Frequency diversity**: replicas of the original signal are provided in the frequency domain. This is applicable in cases where frequency distance is greater than channel coherence bandwidth B_c .



- We have intrinsic frequency diversity when $B > B_c$. This also means selective fading, but in digital systems this can even improve BER.

Diversity (3/5)

- **Time diversity**: replicas of the transmitted signal are provided over time, usually by a combination of channel coding and time interleaving strategies. The key requirement for this form of diversity to be effective is that time distance be greater than channel coherence time T_c .

In such an event, we are assured that two interleaved symbols are transmitted over **uncorrelated** channels.

This technique is not useful for stationary terminals.

CONTINUOUS



Sudden burst of Noise
causing errors

INTERLEAVED



Diversity (4/5)

- **Space/antenna diversity**: This is the most common form of diversity and is an effective method for combating *fast-fading* over space. In this case, replicas of the same transmitted signal are provided using different antennas at the receiver (or at the transmitter). This is applicable in cases where the antenna spacing is larger than the coherence distance L_c to ensure independent fades over different antennas.

In the category of antenna diversity there are two more types of diversity which are peculiar as they do not operate on the “linear space” domain:

- a) **ANGLE DIVERSITY**
- b) **POLARIZATION DIVERSITY**



Diversity (5/5)

a) **Angle diversity:** this technique applies only when the transmitted signals are highly scattered in space (high angle-spread). In such an event the receiver can have two or more highly directional antennas pointing toward totally different directions. This enables the receiver to collect samples of the same signal, from different paths which are independent of each other.

b) **Polarization diversity:** In this type of diversity signals having orthogonal polarizations (e.g: vertical and horizontal) are transmitted by two or more differently polarized antennas and received correspondingly by two or more differently polarized antennas at the receiver. Different polarizations in typical urban environments ensure that there is low fading correlation, moreover with a smaller antenna size wrt traditional antenna diversity. In the best possible case not only are the two polarization branches de-correlated but are also de-coupled, thus allowing *multiplexing* over the same channel.



Rx antenna diversity

In the following we assume frequency-flat fading therefore path delays t_1 to t_N can be replaced by one single delay t_0 (usually $t_0=0$) because:

$$B \approx 1/T_s \ll B_c \approx 1/\Delta t \quad \Rightarrow \quad T_s \gg \Delta t$$

Thus recalling the space and time dependent Channel's impulse response and the low-pass equivalent output signal and including Additive White Gaussian Noise (AWGN) :

$$h(t, \xi, \mathbf{r}) = \sum_i \rho_i \delta(\xi - t_i) f_i(\mathbf{r}) e^{j\{2\pi f_i t - 2\pi f_0 t_i + \vartheta_i\}}$$
$$v(t, \xi, \mathbf{r}) = u(\xi) * h(t, \xi, \mathbf{r}) + n(\xi) = \sum_i \rho_i u(\xi - t_i) f_i(\mathbf{r}) e^{j(2\pi f_i t - 2\pi f_0 t_i + \vartheta_i)} + n(\xi)$$

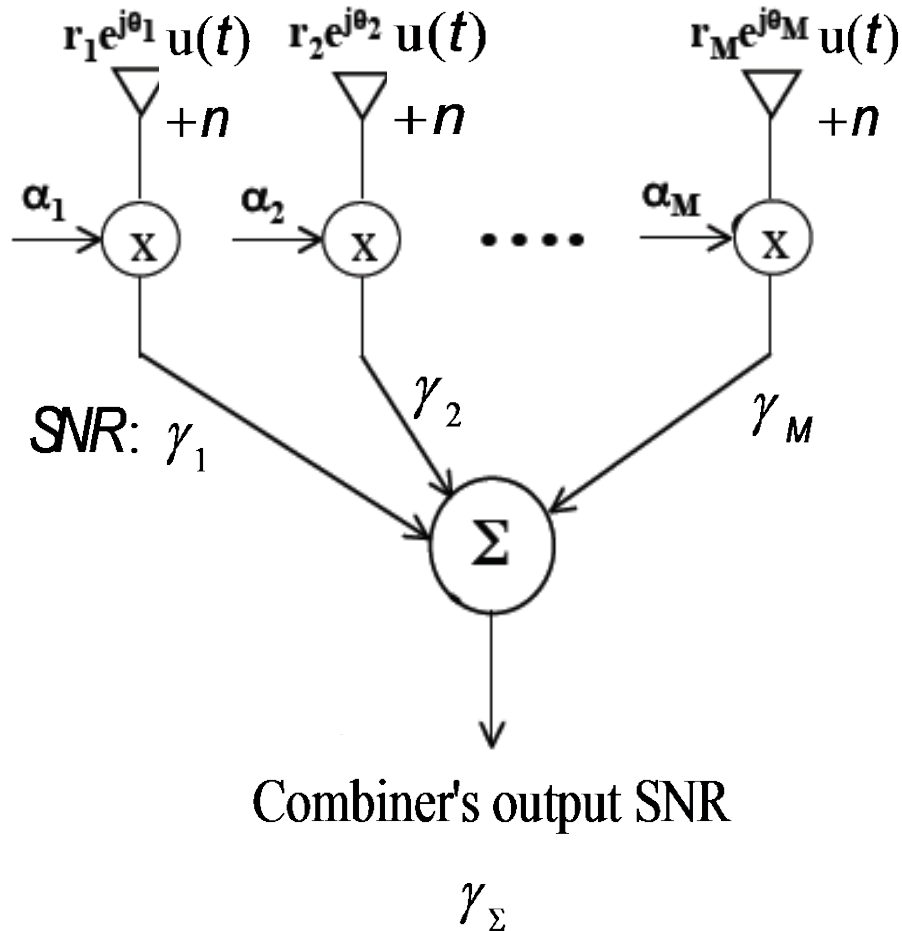
now we have:

$$h(t, \xi, \mathbf{r}) \approx \sum_i \rho_i \delta(\xi - t_0) f_i(\mathbf{r}) e^{j\{2\pi f_i t - 2\pi f_0 t_0 + \vartheta_i\}} = \delta(\xi - t_0) \cdot r(\mathbf{r}, t) e^{j\vartheta(\mathbf{r}, t)}$$
$$v(t, \xi, \mathbf{r}) = \sum_i \rho_i u(\xi - t_0) f_i(\mathbf{r}) e^{j(2\pi f_i t - 2\pi f_0 t_0 + \vartheta_i)} + n(\xi) = u(\xi - t_0) \cdot r(\mathbf{r}, t) e^{j\vartheta(\mathbf{r}, t)} + n(\xi)$$

the channel only multiplies the input signal by a position-dependent complex number and adds noise. This complex number can be considered a random variable (ex: Rayleigh fading, etc.)



Linear diversity combining (M branches)



- we assume antenna spacing $> L_c \rightarrow$ uncorrelated fading branches
- Individual branches are weighted by α_i and summed

Combining more than one branch requires **co-phasing**: the phase of the i -th branch is removed through the multiplication by

$$\alpha_i = a_i e^{-j\theta_i}$$

for some real-valued a_i .

- There are different strategies to choose a_i values – called combining techniques – leading to different SNR's at the combiner's output

Signal to noise ratio

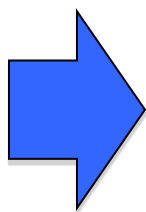
both signal and noise are multiplied by the weights α_i

$$\text{Signal after combining: } v_{\Sigma}(t) = \sum_i v_i(t) = \sum_i a_i r_i u(t) = \alpha_{\Sigma} u(t)$$

$$\text{Received power after combining: } P_R = \alpha_{\Sigma}^2 P_T = \left(\sum_{i=1}^M a_i r_i \right)^2 P_T$$

Noise after combining - same noise spectral power-density N_0 for all branches:

$$N = \sum_i a_i^2 N_0 B$$



$$\text{SNR after combining } \gamma_{\Sigma} = \frac{P_R}{N} = \frac{\left(\sum_{i=1}^M a_i r_i \right)^2 P_T}{\left(\sum_{i=1}^M a_i^2 \right) N_0 B}$$

$$\text{Before combining at the } i\text{-th branch: } \gamma_i = \frac{P_{R_i}}{N} = \frac{r_i^2 P_T}{N_0 B}$$

Linear combining techniques

The main combining techniques are:

1) **Selection combining (SC)**. In SC, the combiner outputs the signal on the branch with the highest SNR $\gamma_{i\text{MAX}}$. This is equivalent to choosing the branch with the highest received signal power r_i^2 if the noise power is the same on all branches. Since only one branch is used at a time, SC often requires just one receiver that is switched into the active antenna branch. Since only one branch output is used, co-phasing of multiple branches is not required.

2) **Equal gain combining (ECG)**. In ECG, the signals on each branch are co-phased and then combined with equal weighting ($\alpha_i = e^{-j\theta}$). The SNR of the combiner output is given by

$$\gamma_{\Sigma} = \frac{1}{M} \left(\sum_{i=1}^M r_i \right)^2 \frac{P_T}{N_0 B}$$

3) **Maximum ratio combining (MRC)**. MRC is the technique which maximizes the SNR of the combiner output. Intuitively, branches with a high SNR should be weighted more than branches with a low SNR: therefore the weights should be proportional to the branch SNRs. The signals on each branch are co-phased and then combined with the optimal weights $\mathbf{a}_i = r_i / \sqrt{N_0}$. The combiner SNR is:

$$\gamma_{\Sigma} = \sum_{i=1}^M r_i^2 \frac{P_T}{N_0 B} = \sum_{i=1}^M \gamma_i \quad (\text{The SNR of the combiner output is the sum of SNRs on each branch})$$



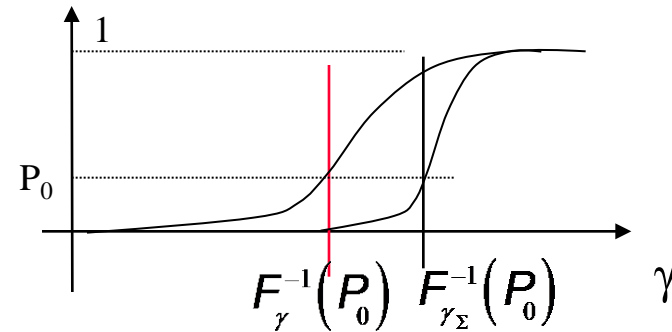
Diversity Gain

If the fading is considered random, then the SNR is also a random variable.

A diversity performance measure is Diversity Gain (DG)

If $F_{\gamma}(\gamma)$ and $F_{\gamma_{\Sigma}}(\gamma)$ are the cumulatives of the SNR before and after combining, and P_0 is a given outage probability, then:

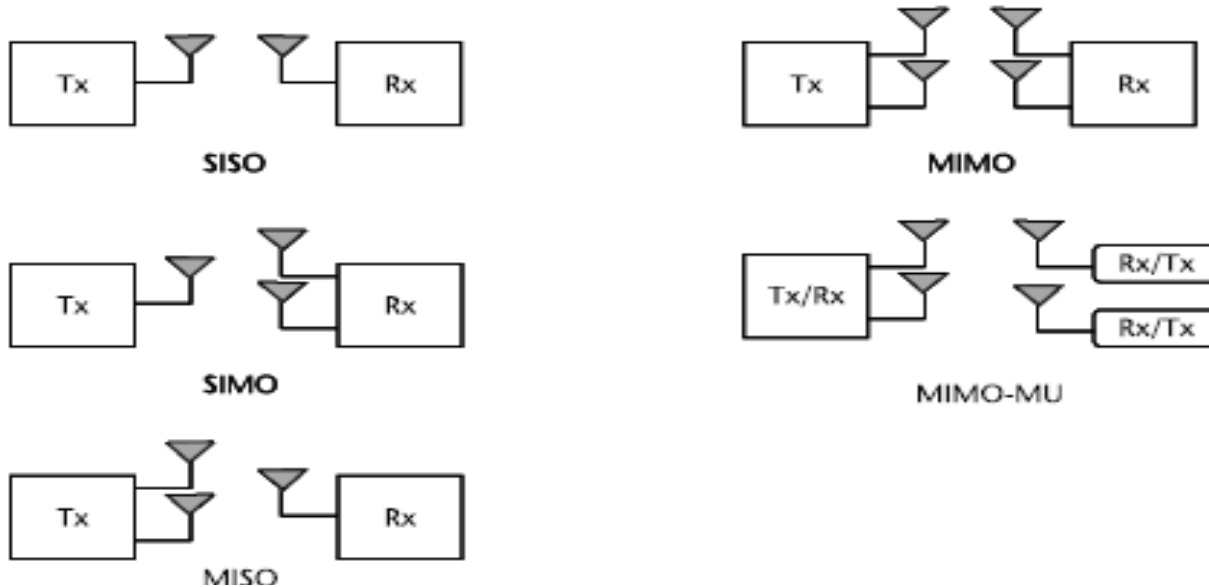
$$DG = 20 \text{Log} \left\{ \frac{F_{\gamma_{\Sigma}}^{-1}(P_0)}{F_{\gamma}^{-1}(P_0)} \right\}$$



The DG depends on the combining technique and on the residual fading correlation between the different branches. If the branches are correlated then $F_{\gamma}(\gamma)$ and $F_{\gamma_{\Sigma}}(\gamma)$ are the same and there is no gain

Multiantenna (MIMO) Systems

- **Single Input - Single Output (SISO)** is the well-known wireless configuration
- **Single input - Multiple Output (SIMO)** uses a single transmitting antenna and multiple (N_R) receive antennas
- **Multiple input - Single Output (MISO)** has multiple (N_T) transmitting antennas and one receive antenna
- **Multiple input - Multiple Output (MIMO)** has multiple (N_T) transmitting antennas and multiple (N_R) receive antennas
- **MIMO-multiuser (MIMO-MU)** refers to a configuration that comprises a base station with multiple transmit/receive antennas interacting with multiple



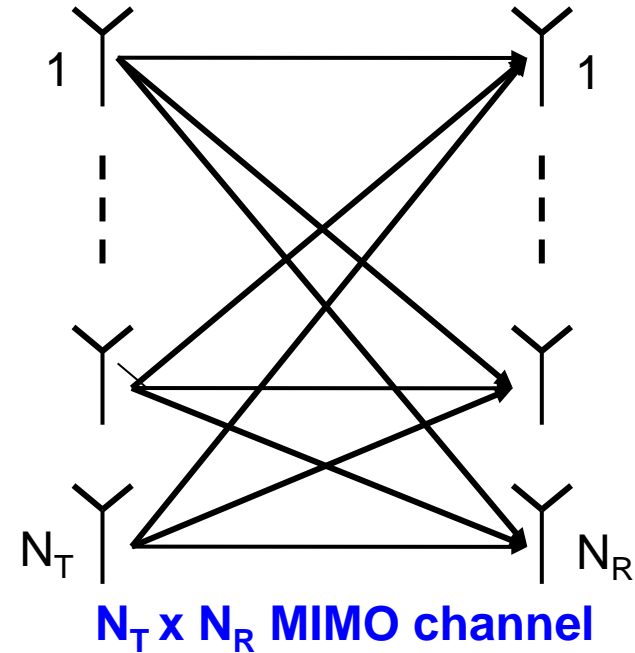
The MIMO channel $N_T \times N_R$

In a MIMO systems, the transmitted signal $u(\xi, \mathbf{r})$ is replaced by the $N_T \times 1$ vector $\mathbf{u}(\xi)$:

$$\mathbf{u}(\xi) = \begin{bmatrix} u(\xi, \mathbf{r}_1^{Tx}) \\ \vdots \\ u(\xi, \mathbf{r}_{N_T}^{Tx}) \end{bmatrix} = \begin{bmatrix} u_1(\xi) \\ \vdots \\ u_{N_T}(\xi) \end{bmatrix}$$

whereas $v(t, \xi, \mathbf{r})$ and $n(t, \xi, \mathbf{r})$ are replaced by the $N_R \times 1$ vectors $\mathbf{v}(t, \xi)$ and $\mathbf{n}(t, \xi)$:

$$\mathbf{v}(t, \xi) = \begin{bmatrix} v(t, \xi, \mathbf{r}_1^{Rx}) \\ \vdots \\ v(t, \xi, \mathbf{r}_{N_R}^{Rx}) \end{bmatrix} = \begin{bmatrix} v_1(t, \xi) \\ \vdots \\ v_{N_R}(t, \xi) \end{bmatrix} \quad \mathbf{n}(t, \xi) = \begin{bmatrix} n(t, \xi, \mathbf{r}_1^{Rx}) \\ \vdots \\ n(t, \xi, \mathbf{r}_{N_R}^{Rx}) \end{bmatrix} = \begin{bmatrix} n_1(t, \xi) \\ \vdots \\ n_{N_R}(t, \xi) \end{bmatrix}$$



Note: usually noise is the same for all antennas and constant over t



The MIMO channel $N_T \times N_R$ (2)

The time-varying channel impulse response is replaced by the $N_R \times N_T$ **channel matrix** $\mathbf{H}(t, \xi)$, given by:

$$\mathbf{H}(t, \xi) = \begin{bmatrix} h_{11}(t, \xi) & \dots & h_{1N_T}(t, \xi) \\ \vdots & \ddots & \vdots \\ h_{N_R1}(t, \xi) & \dots & h_{N_R N_T}(t, \xi) \end{bmatrix}$$

So that the input output relation can be expressed in the matrix form:

$$\mathbf{v}(t, \xi) = \mathbf{H}(t, \xi) * \mathbf{u}(\xi) + \mathbf{n}(\xi) \hat{=} \int_{\tau} \mathbf{H}(t, \xi) \mathbf{u}(\xi - \tau) d\tau + \mathbf{n}(\xi)$$

The MIMO channel $N_T \times N_R$ (3)

If the channel is *time-invariant*, the dependence of channel matrix on t vanishes: then we can resort to one single time scale, t : $\mathbf{H}(\cdot) = \mathbf{H}(t)$. If the channel furthermore is *frequency-flat*, i.e. $B \ll B_C$ the channel matrix can be considered non-zero only for $t=0$ [$\mathbf{H}(t) = \mathbf{H}^{\text{TM}}(t)$] and hence the input-output relation boils down to:

$$\mathbf{v}(t) \cong \mathbf{H} \cdot \mathbf{u}(t) + \mathbf{n}(t)$$

$u_j(t)$, h_{ij} , $v_i(t)$, $n_i(t)$ are complex-valued, and can be modeled as stochastic processes. If the time parameter t is fixed ($t = t_0$), they become **complex random variables**. Assuming scalar linear modulation (PAM or QAM), we have for the i -th transmitted signal:

$$u_i(t) = \sum_k s_i[k] D[t - kT_s]$$

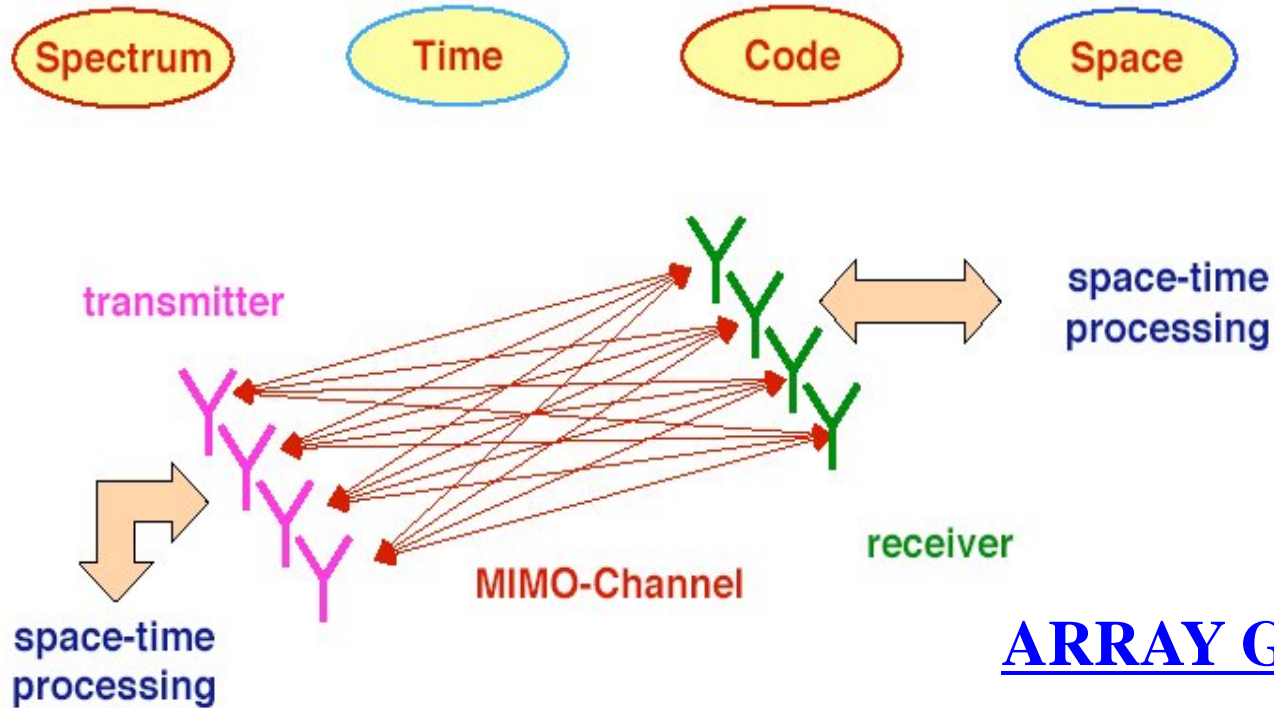
the time-discrete model can be derived if signals are sampled at instants $t = kT_s + \Delta$, where T_s is the symbol duration ($1/T_s \approx B$ transmission bandwidth), and Δ the sampling delay, we obtain a discrete frequency-flat MIMO I/O relation:

$$\mathbf{v}[k] = \mathbf{H} \mathbf{u}[k] + \mathbf{n}[k] \quad k = 0, 1, 2, \dots$$



Main advantages of MIMO systems

Efficient use of the 4-D communication space:



ARRAY GAIN

DIVERSITY GAIN

MULTIPLEXING GAIN

Array Gain

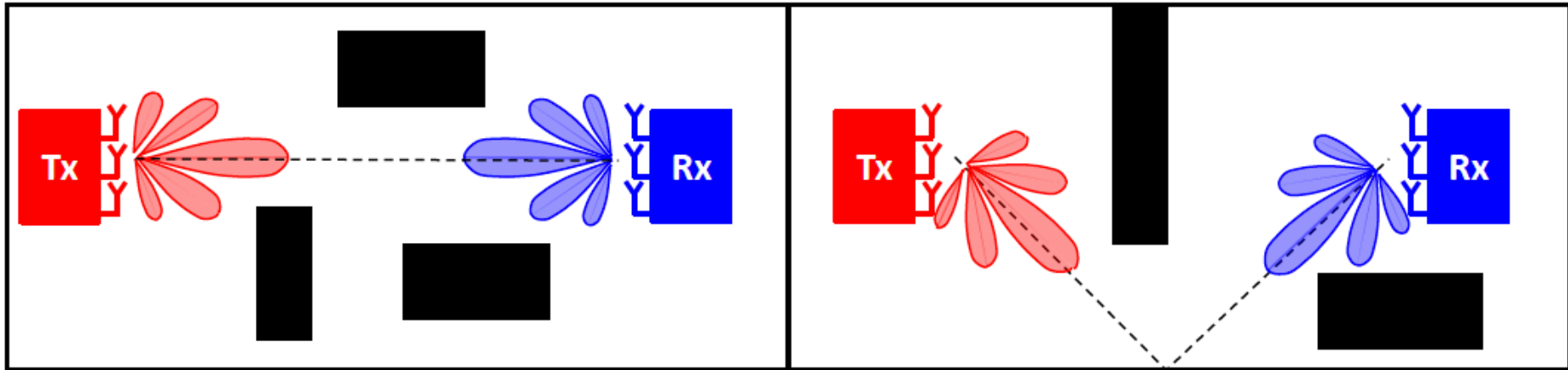
Array gain (or **beamforming gain**): is the average **increase in the signal-to-noise ratio (SNR)** at the receiver that arises from the coherent combining effect of multiple antennas at the receiver or transmitter or both, if the antennas are used as phased arrays.

- TX array gain: if **Channel State Information (CSI)** are known to the **multiple-antenna transmitter** the transmitter will weight the TX signals, depending on the channel coefficients, so that there is coherent summing at the receiver (MISO case)
- RX array gain: if we have only one antenna at the transmitter (SIMO case) and a multiple antenna receiver, which has perfect knowledge of the CSI, then the receiver can suitably weight the incoming signals at the antenna elements so as to increase the signal level.
- MIMO array gain: basically, multiple antenna systems require perfect CSI knowledge at the transmitter and at the receiver to achieve maximum MIMO array gain



Beamforming

- The radiating elements at the Tx / Rx can be regarded as an 'antenna array', whose radiation pattern can be effectively shaped and steered by properly setting the amplitudes and phase relations among the signals sent to / coming from the different elements;



- Of course, the main radiation lobes should be always oriented in (or at least around) the directions of departure / arrival of the 'dominant' path, i.e. the less attenuated one (therefore carrying the highest field intensity);
- With respect to the SISO case (Tx & Rx equipped with just 1 radiating element) with the same overall radiated power, the power carried by the dominant path is increased and the multipath fading is also reduced \Rightarrow the final outcome is a SNR increase

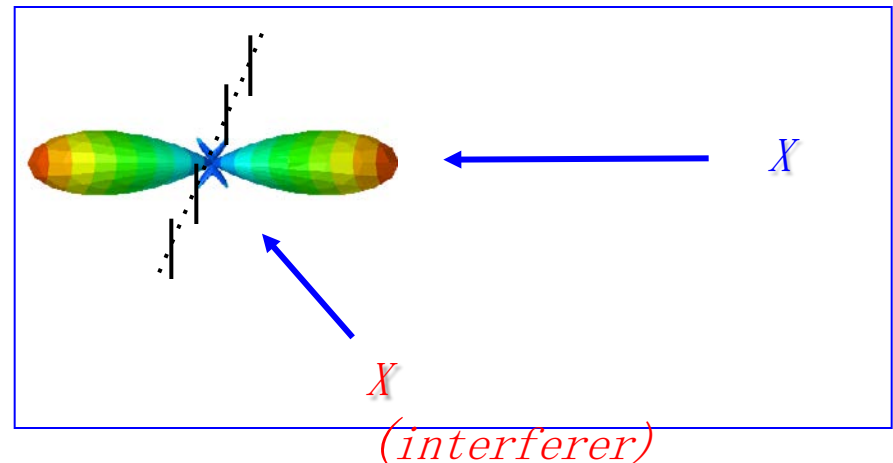
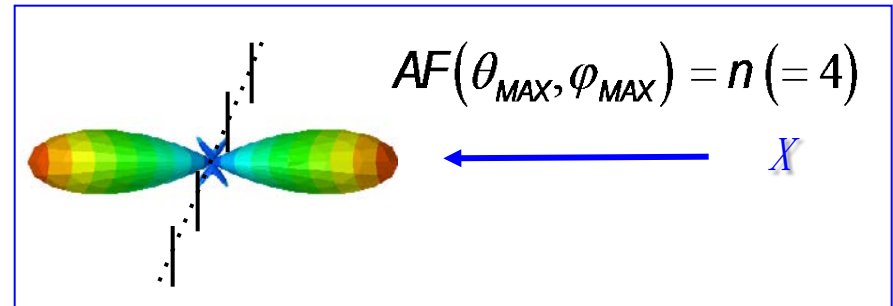
Array Gain example

With *beamforming* it is possible to increase SNR and SIR

- Higher directivity

$$G_{array}(\theta, \varphi) = AF(\theta, \varphi) \cdot G_o(\theta, \varphi)$$

- Space Division (SDMA)



Diversity gain

MIMO naturally implements antenna/space diversity.

In particular we can have:

1) **RECEIVE DIVERSITY**: (MIMO and SIMO) The received signal from different antennas (branches) is properly combined at the RX using one of the cited combining techniques

2) **TRANSMIT DIVERSITY**: (MIMO and MISO) In basic Tx-diversity schemes, the same signal is transmitted from multiple antennas at the Tx.

In general, controlled redundancies at the transmitter are introduced (pre-coding), which can be then exploited by appropriate signal processing techniques at the receiver. Generally this technique requires complete CSI at the transmitter.



Receive Diversity & Diversity order

- In RX antenna diversity, the receiver that has multiple antennas receives multiple replicas of the same transmitted signal, assuming that the transmission came from the same source (SIMO).
- The intrinsic diversity potential of the channel is related to multipath richness which produces low correlation, i.e. low L_C . L_C is also related to the angle spread.

The higher multipath richness, the lower L_C . With only one path $L_C = \infty$.

- Diversity is characterized by **the number of independent fading branches**, also known as ***diversity order***.
- In SIMO channels if antenna spacing $> L_C$ (correlation length) the system achieves ***full diversity order*** M , where M is the number of RX antennas



TX diversity & Space-Time Coding

- **Space-time codes** exploit diversity across space and time (both time diversity and spatial diversity are used).
- With the advent of space-time coding schemes (ex. Alamouti's scheme), it became possible to implement transmit diversity **without knowledge of the channel**. This was one of the fundamental reasons why the MIMO concept became popular.
- Generally, space-time codes for MIMO exploit both transmit as well as receive diversity schemes, yielding a high quality of reception.

EXAMPLE: THE ALAMOUTI'S SCHEME

Alamouti's scheme is designed for a 2x1 MISO system (two-antenna transmit diversity). The scheme works over two symbol periods where it is assumed that the channel gain is constant. Over the 1st time slot two different symbols s_1 and s_2 each with energy $E_s/2$ are transmitted simultaneously from antennas 1 and 2, respectively. Over the next time slot symbol $-s_2^*$ is transmitted from antenna 1 and symbol s_1^* is transmitted from antenna 2. Let's Assume complex channel gains $h_i = r_i e^{j\theta_i}$ $i=1,2$ between the i -th TX antenna and the RX antenna. The received symbol over the first time slot is $y_1 = h_1 s_1 + h_2 s_2 + n_1$ while the received symbol over the second time slot is $y_2 = -h_1 s_2^* + h_2 s_1^* + n_2$

[$n_i, i=1,2$ is the AWGN noise sample at the RX associated with the i -th symbol transmission] **23**



Alamouti's scheme (continued)

- The Rx then conjugates y_2 , so that, using matrix notation, we have:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} = \mathbf{H}_A \mathbf{s} + \mathbf{n}$$

- Let us define the new received vector $\mathbf{z} = \mathbf{H}_A^H \mathbf{y}$, where $\mathbf{H}_A^H \triangleq (\mathbf{H}_A^*)^T$ (hermitian conjugate) The structure of \mathbf{H}_A implies that

$$\mathbf{H}_A^H \mathbf{H}_A = \begin{pmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{pmatrix}_2$$

N.B. RX knows the channel, and it can pre-multiply \mathbf{y} with \mathbf{H}_A

is diagonal, and thus

$$\mathbf{z} = \begin{bmatrix} z_1 & z_2 \end{bmatrix}^T = \begin{pmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{pmatrix}_2 \mathbf{s} + \tilde{\mathbf{n}}$$

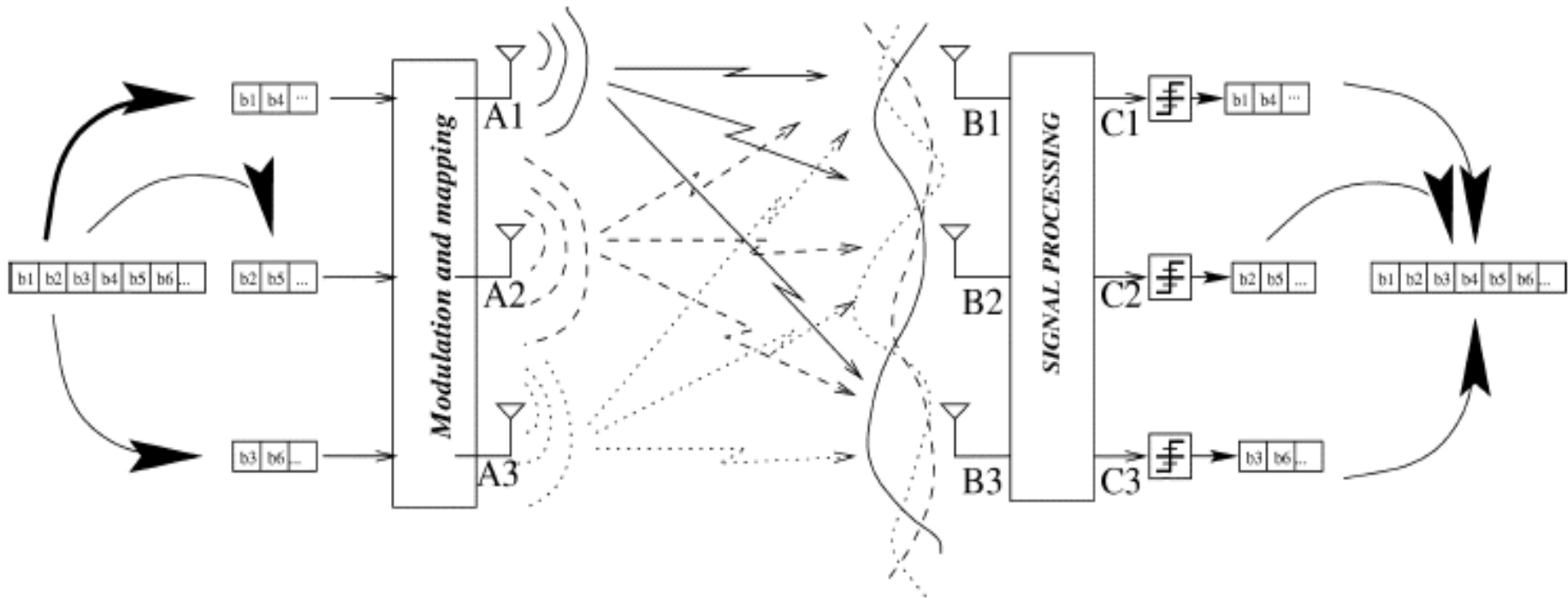
The diagonal nature of \mathbf{z} effectively decouples the 2 symbol transmissions, so that each component of \mathbf{z} corresponds to one of the transmitted symbols:

$$z_i = \left(|h_1|^2 + |h_2|^2 \right) s_i + n_i, \quad i=1,2.$$

- The received SNR corresponds to the SNR for z_i given by $\gamma_i = \left(|h_1|^2 + |h_2|^2 \right) E_s / 2N_0$ and is thus equal to the sum of SNRs on each branch (as in MRC!). Thus, **the Alamouti's scheme achieves a diversity order of 2**, the maximum possible for a 2-antenna transmit system, despite the fact that TX does not know the channel!

Multiplexing gain

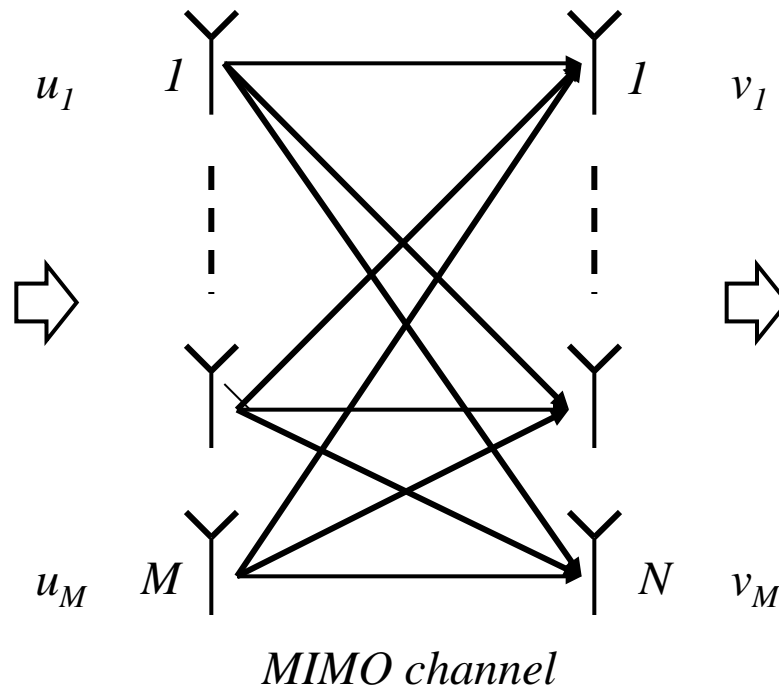
- **Spatial multiplexing** offers a linear (in the number of transmit-receive antenna pairs or $\min\{N_R, N_T\}$) increase in the transmission rate (or capacity) for the same bandwidth and with no additional power expenditure.
- It is **only** possible in MIMO channels.
- Why? Because it is possible to exploit the multipath to carry different information streams



Multiplexing Gain: basics

- For Ex. The symbol-stream is split into M parallel symbol-streams transmitted simultaneously

Ex: time-discrete representation of a MIMO channel



MIMO channel matrix

$$\mathbf{H} = \begin{bmatrix} h_{11} & \cdot & \cdot & h_{1M} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ h_{N1} & \cdot & \cdot & h_{NM} \end{bmatrix}$$

$$\mathbf{v} = \mathbf{H} \cdot \mathbf{u} (+\mathbf{n})$$

Multiplexing Gain: basics

- How to reconstruct the input vector \mathbf{u} from the output vector \mathbf{v} ?

$$\mathbf{v} = \mathbf{H} \cdot \mathbf{u} (+\mathbf{n})$$

- If \mathbf{H} has full rank it is possible to solve the linear system: $\mathbf{v} \rightarrow \mathbf{u}$
- It must be $N \geq M$
- Noise need to be low (high SNR)
- What determines \mathbf{H} rank? The multipath radio channel !

\mathbf{H} rank \leftrightarrow # independent paths & # antennas \leftrightarrow # spatial degrees of freedom

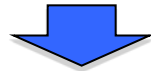
CSI at the Rx is necessary !



Example: MIMO nxn

$$\mathbf{v} = \mathbf{H} \cdot \mathbf{u} (+\mathbf{n})$$

- If the CSI, i.e. the full channel matrix is known at the transmitter then

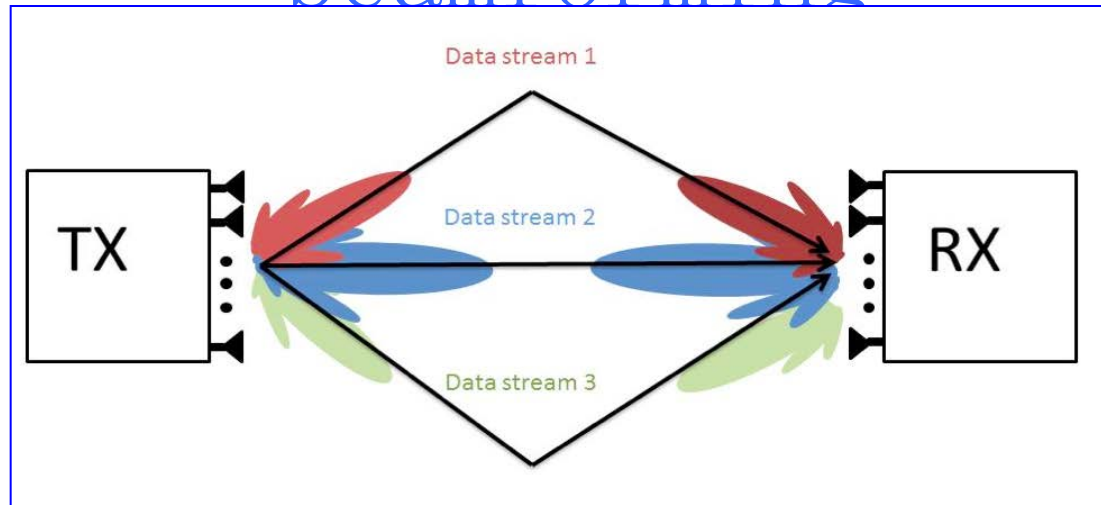


$$\mathbf{H}^{-1} \cdot \mathbf{v} = \mathbf{H}^{-1} \cdot \mathbf{H} \cdot \mathbf{u} + \mathbf{H}^{-1} \mathbf{n} = \mathbf{u} + \mathbf{H}^{-1} \mathbf{n} \approx \mathbf{u}$$

It is possible to reconstruct the n symbols transmitted simultaneously if the noise term is not too high...

In this case multiplexing gain is n as n symbols are transmitted over the channel instead of 1 of the SISO case.

Multiplexing through beamforming



Here **beamforming** is enforced to multiplex different data-streams over different paths

From this example it is evident that the **degrees of freedom** of the channel is equal to the number of **independent paths**

It is not feasible 'cause it is very difficult to match the antenna pattern to real-life multipath with simple antennas

Moreover CSI at the Tx AND at the Rx would be necessary



Multiplexing reality

In real systems multiplexing is achieved through proper **space-time processing** (or **signal processing**)

Beamforming is a particular signal processing technique

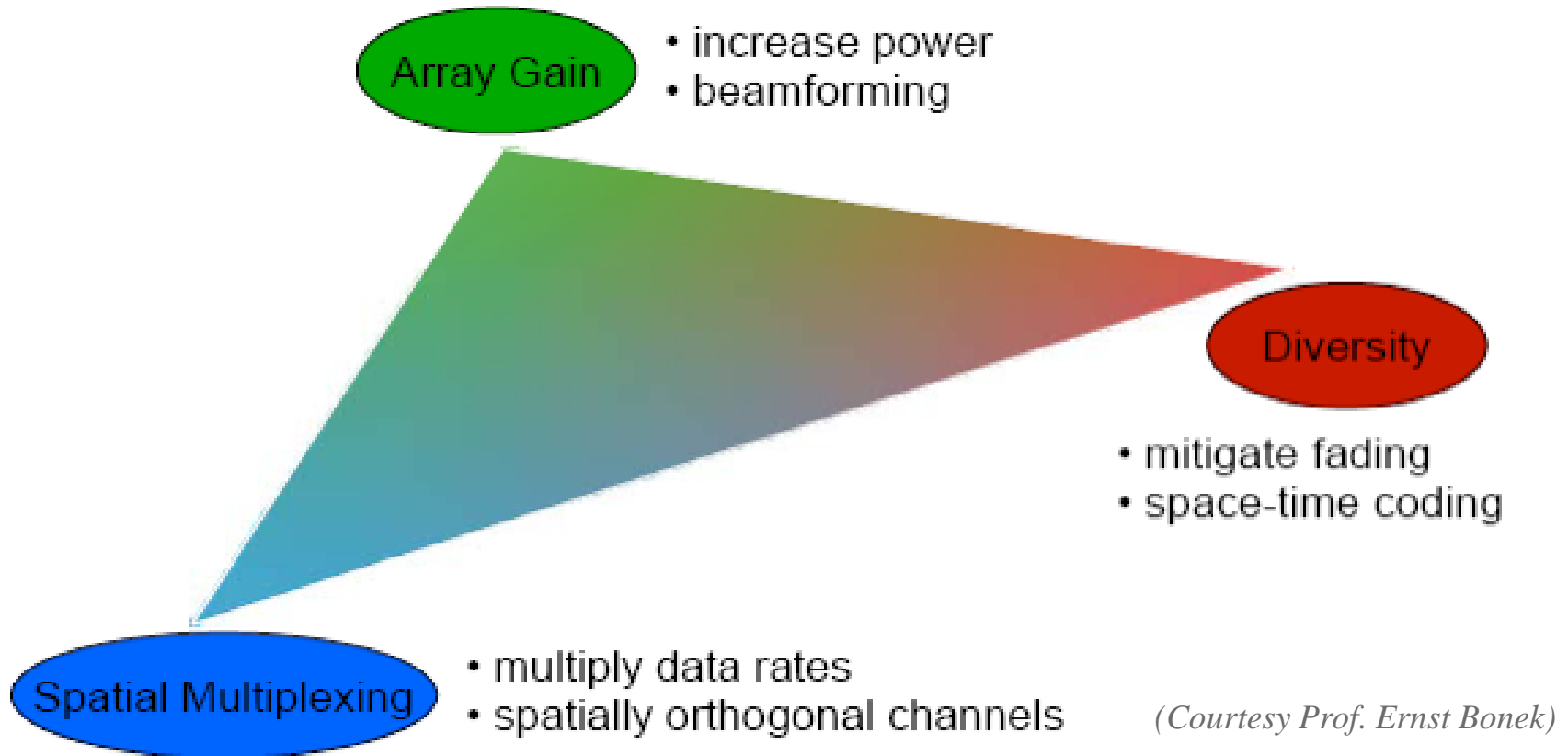
Precoding at the Tx and corresponding **decoding** at the Rx are necessary

To achieve full multiplexing gain complete CSI at both ends is necessary: the Rx must “sense” the channel characteristics and transmit them to the Tx through a **feedback loop**...problems with rapidly varying channels

With pure multiplexing there is no diversity gain: pure multiplexing and pure diversity are mutually exclusive



Multiplexing – diversity trade-off



How to implement a MIMO transmission technique?

Through multidimensional knowledge of the propagation channel !

References

- [1] A. J. Paulraj, R.U. Nabar and D. A. Gore, “*Introduction to Space-Time Wireless Communications*”, Cambridge Univ. Press, 2003.
- [2] A. Goldsmith, *Wireless Communications*, Cambridge University Press, 2005.
- [3] D. Gesbert, M. Shafi, D. Shiu, P. J. Smith, A. Naguib, “*From Theory to Practice: An Overview of MIMO Space-Time Coded Wireless Systems*”, IEEE J. Select. Areas Commun., vol. 21, no. 3, April 2003.
- [4] G. J. Foschini, M. J. Gans, “*On limits of wireless communications in a fading environment when using multiple antennas*”, Wireless Pers. Commun., vol. 6, pp. 311-335, Mar. 1998.
- [5] I. E. Telatar, “*Capacity of multiantenna gaussian channels*”, Eur. Trans. Tel., vol. 10, no. 6, pp. 585-595, Nov./Dec. 1999.

