C – IL CANALE RADIOMOBILE

- Caratterizzazione deterministica del canale radiomobile
  - Fuzioni di trasferimento del canale: caso statico e dinamico
  - Fading piatto e selettivo
  - I parametri sintetici (delay-spread, banda di coerenza ecc.).
  - Estensione al dominio spaziale
  - Autocorrelazioni
  - Esempi
- Tecniche di diversità, MIMO, e space-time coding
  - Tecniche di diversità
  - Matrice di canale e MIMO
  - Multiplexing gain e cenni a space-time coding.

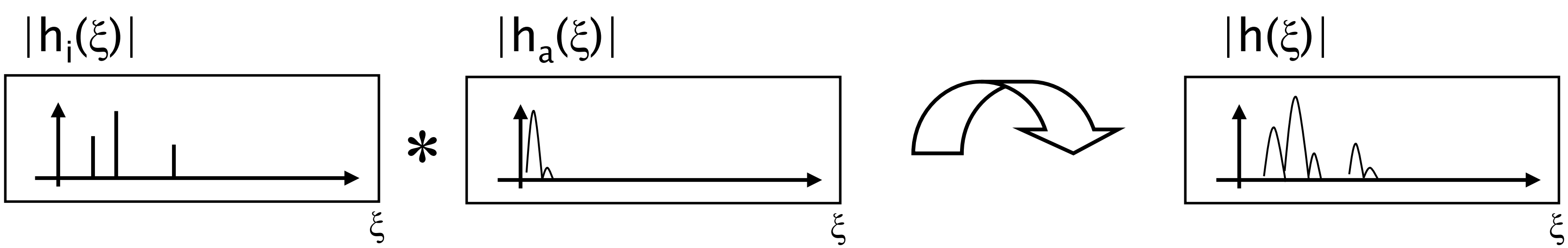
Synthetic channel parameters (1/4)

Let's consider a fixed sounding time t=0, therefore we get $h(0,\xi)=h(\xi)$.

Notice that $h(\xi)$ is not a sequence of Dirac's impulses in real life, 'cause real channels have a limited bandwidth:

$$h(\xi) = h_i(\xi) * h_a(\xi)$$

Es:



where:

$h_i(\xi)$ ideal impulse response of the propagation channel

$h_a(\xi)$ impulse response of the antennas, amplifiers and mo-dem

Power-delay profile

The normalized **power-delay profile** can be defined as:

$$p(\xi) = \frac{|h(\xi)|^2}{\int |h(\xi)|^2 d\xi} \quad [\text{W/s}]; \qquad \text{it's normalized: } \int p(\xi) d\xi = 1$$

If a statistical evaluation of p(ξ) in a given environment is needed, then averaging M different realizations/samples of p(ξ) we get:

$$\overline{p}(\xi) \approx \frac{1}{M} \sum_{k=1}^{M} p^k(\xi) \qquad \text{(average power-delay profile)}$$

Power-delay profile (alternative definition)

$$h(\xi) \qquad\qquad\qquad\qquad p(\xi)$$ (time power-spectrum)

$$\mathbf{F}\downarrow \qquad\qquad\qquad\qquad \uparrow \mathbf{F}^{-1}$$

$$H(f) \longrightarrow C(f)$$ (frequency correlation)

Power-Doppler profile

Similarly, at a frequency f=0, using F(ν,0)=F(ν) the normalized **power-Doppler profile** can be defined so that:

$$p_\nu(\nu) = \frac{|F(\nu)|^2}{\int |F(\nu)|^2 d\nu} \quad \text{[W/Hz]} \qquad \qquad \overline{p}_\nu(\nu) \approx \frac{1}{M}\sum_{k=1}^{M} p_k^\nu(\nu)$$

Power-profiles, also called power-densities or power spectra can be defined on domains were the transfer function is a (limited) energy-signal (i.e. δ-kind in the discrete case).

In our case therefore we have power profiles in the excess delay and in the Doppler frequency domains

Dispersion parameters

RMS delay spread (DS)

$$DS = \sqrt{\int p(\xi)(\xi - T_{M0})^2 \, d\xi} \qquad T_{M0} = \int p(\xi)\xi \, d\xi$$

RMS Doppler spread (*W*)

$$W = \sqrt{\int p_\nu(\nu)(\nu - W_0)^2 \, d\nu} \qquad W_0 = \int p_\nu(\nu)\nu \, d\nu$$

DS and W are nothing but the standard deviations of p and p_ν interpreted as pdf's.

An estimate of coherence bandwidth and coherence time can be derived:

Coherence bandwidth

$$B_c \simeq \frac{1}{DS}$$

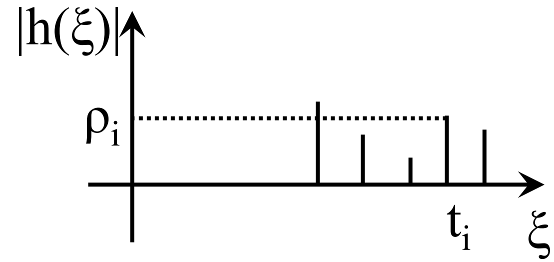
Coherence time

$$T_c \simeq \frac{1}{W}$$

Usually, average versions of DS, W, B_c, T_c are derived using the corresponding average power profiles.

Discrete case

$$h(\xi)=\sum_{i=1}^{N_r}\rho_i\,\delta(\xi-t_i)\,e^{j(-2\pi f_0\cdot t_i+\theta_i)}$$



Since h(ξ) is a discrete function which is defined only for $\xi=\{t_i\}$, and therefore the impulses do not overlap, we have:

$$|h(\xi)|=\left|\sum_{i=1}^{N_r}\rho_i\,\delta(\xi-t_i)\,e^{j(-2\pi f_0\cdot t_i+\theta_i)}\right|=\sum_{i=1}^{N_r}\left|\rho_i\,\delta(\xi-t_i)\,e^{j(-2\pi f_0\cdot t_i+\theta_i)}\right|=\sum_{i=1}^{N_r}|\rho_i|\,\delta(\xi-t_i)$$

then

$$|h(\xi)|^2=\left(\sum_{i=1}^{N_r}|\rho_i|\cdot\delta(\xi-t_i)\right)\cdot\left(\sum_{i=1}^{N_r}|\rho_i|\cdot\delta(\xi-t_i)\right)=\sum_{i=j=1}^{N_r}\rho_i^2\cdot\delta(\xi-t_i)$$

were the last equal sign is due to the fact that the double products at the left-hand side are non-zero only when i=j. Also, for simplicity we have assumed that $\delta^2(\xi-t_i)=\ \delta(\xi-t_i)$

Discrete case – Power profiles

Therefore we get the power-deplay profile:

$$p(\xi)=\frac{\sum_{i=1}^{N}\rho_i^2\,\delta(\xi-t_i)}{\int\sum_{i=1}^{N}\rho_i^2\,\delta(\xi-t_i)\,d\xi}=\frac{\sum_{i=1}^{N}\rho_i^2\,\delta(\xi-t_i)}{\sum_{i=1}^{N}\rho_i^2\int\delta(\xi-t_i)\,d\xi}\Rightarrow\ p(\xi)=\frac{\sum_{i=1}^{N}\rho_i^2\,\delta(\xi-t_i)}{\sum_{i=1}^{N}\rho_i^2}$$

And with a similar procedure we can get the power-Doppler profile:

$$p_v(v)=\frac{\sum_{i=1}^{N}\rho_i^2\,\delta(v-f_i)}{\sum_{i=1}^{N}\rho_i^2}\quad \text{(power-Doppler profile)}$$

Of course these profiles can be averaged to get estimates of the corresponding average functions.

Discrete case - dispersion parameters

Discrete DS and W can be directly derived from the power-delay and power-Doppler profiles:

$$TM_0 = \sum_{i=1}^{N} t_i \cdot p_i$$

$$DS = \sigma_\xi = \sqrt{\sum_{i=1}^{N} \left(t_i - TM_0\right)^2 \cdot p_i}$$

where: $p_i = \frac{\rho_i^2}{P_{TOT}} = \frac{\rho_i^2}{\sum_{i=1}^{N} \rho_i^2}$

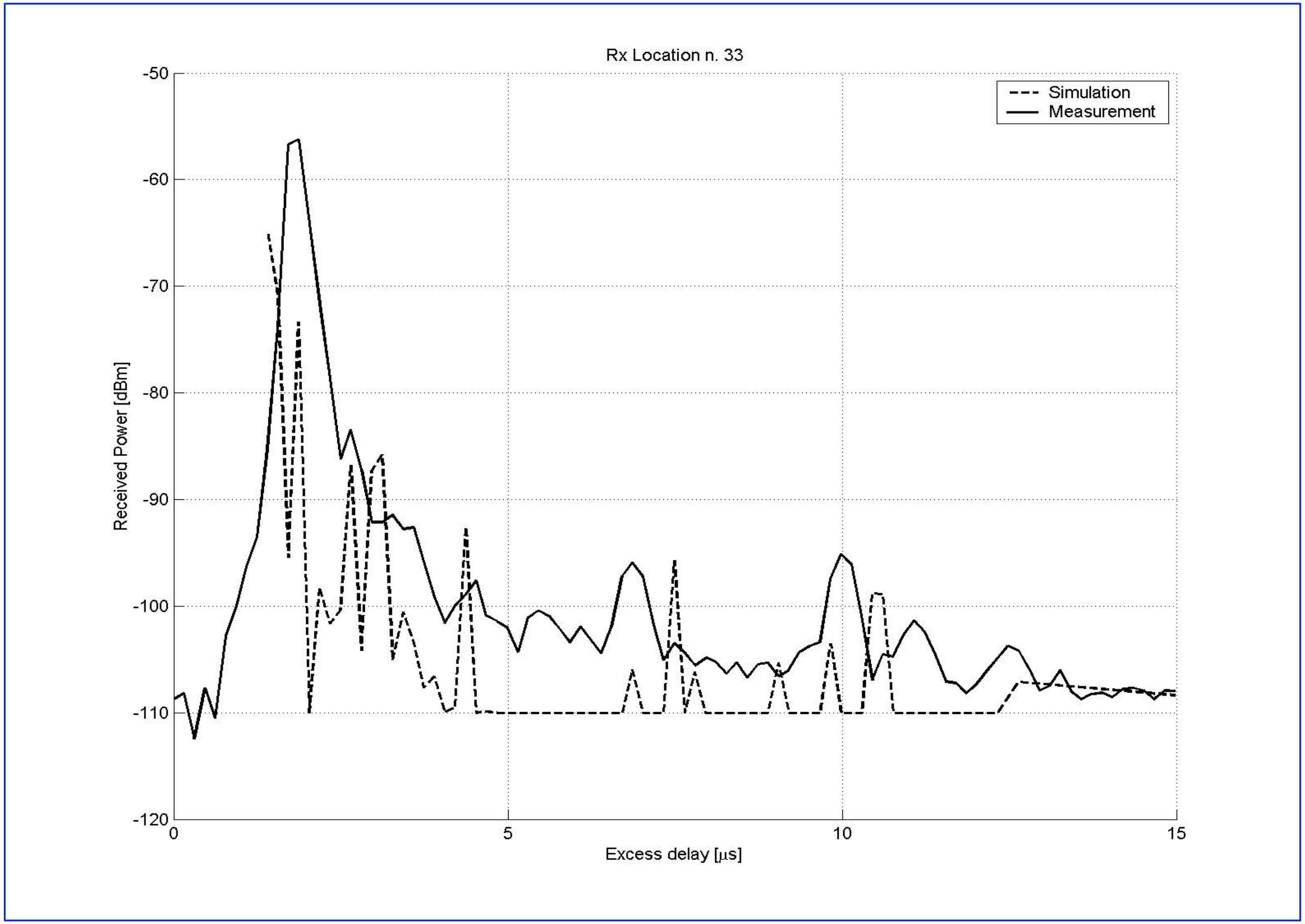
And:

$$W_0 = \sum_{i=1}^{N} f_i \cdot p_i$$

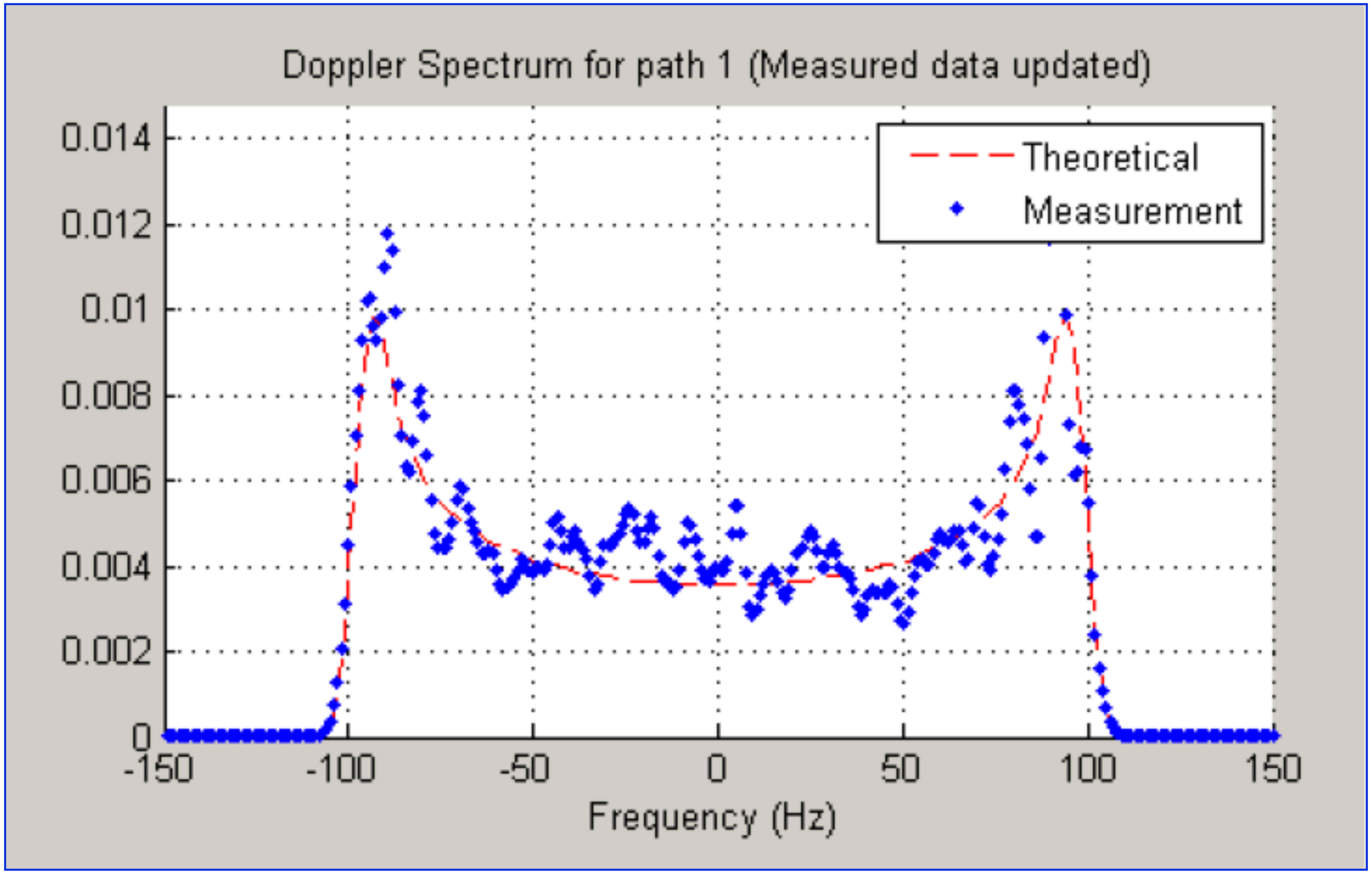
$$W = \sigma_\nu = \sqrt{\sum_{i=1}^{N} \left(f_i - W_0\right)^2 \cdot p_i}$$

Other parameters can be introduced which refer to space instead of excess delay or doppler frequency. Thus a so called ***multidimensional characterization of the mobile radio channel*** can be derived, in both a statistical (through statistical/average functions) and deterministic way (measured or simulated data).

Example of power-delay profile



Example of power-Doppler profile



What is multidimensional propagation characterization?

- Radio propagation characterization in terms of coverage, path-loss, path-gain or received power is usually called ***narrowband characterization***

- Radio propagation characterization in terms of power-delay profile, delay spread, power-Doppler profile, Doppler spread, Coherence bandwidth or Coherence time, frequency response, etc. is usually called ***wideband characterization***

- Radio propagation characterization in terms of all of the previous parameters and also in terms of spatial parameters (angle of arrival/emission, power-angle profiles, angle spread, etc.) is called ***multidimensional characterization***

- ***In short: multidimensional characterization is the characterization of multipath propagation with respect to all domain dimensions: amplitude, time, frequency, Doppler frequency, space***

⇩

MIMO

Extension to the angle domain (1/2)

Let’s consider the low-pass channel impulse response:

$$h(t,\xi) = \sum_i \rho_i \delta\left[\xi - t_i\right] e^{j\left\{2\pi f_i t - 2\pi f_0 t_i + \vartheta_i\right\}}$$

If, for instance, the azimuth angle of arrival φ is considered then since the signal arrives from discrete angles φ_i corresponding to paths, it is straightforward to get an angle-dependent version of h:

$$h(t,\xi,\phi) = \sum_i \rho_i \delta\left[\xi - t_i\right] \delta\left[\phi - \phi_i\right] e^{j\left\{2\pi f_i t - 2\pi f_0 t_i + \vartheta_i\right\}}$$

Of course the same Dirac’s impulse-dependance also appears in the other transfer functions, e.g:

$$H(t,f,\phi) = \sum_i \rho_i \delta\left[\phi - \phi_i\right] e^{j\left\{2\pi f_i t - 2\pi\left(f + f_0\right) t_i + \vartheta_i\right\}}$$

Extension to the angle domain (2/2)

If H(φ)=H(0,0, φ), then a **power-azimuth profile** can be defined:

$$p_\phi(\phi) = \frac{|H(\phi)|^2}{\int |H(\phi)|^2 \, d\phi} \; ;$$

In the ideal case the power-azimuth profile has the simple form:

$$|H(\phi)| = \sum_{i=1}^{N} \rho_i \, \delta(\phi - \phi_i) \;\Rightarrow\; p_\phi(\phi) = \frac{\sum_{i=1}^{N} \rho_i^2 \, \delta(\phi - \phi_i)}{\sum_{i=1}^{N} \rho_i^2} = \sum_{i=1}^{N} p_i \, \delta(\phi - \phi_i)$$

Similarly, a **power-elevation profile** can be derived. Through the power-azimuth profile the Azimuth-Spread (AS) can be defined

Azimuth-Spread definition

Mean angle (azimuth) of arrival:

$$\bar{\phi} = \int_0^{2\pi} \phi \, p_\phi(\phi) \, d\phi \quad \text{with} \;\; p_\phi(\phi) = power - azimuth\ profile$$

RMS Azimuth Spread:

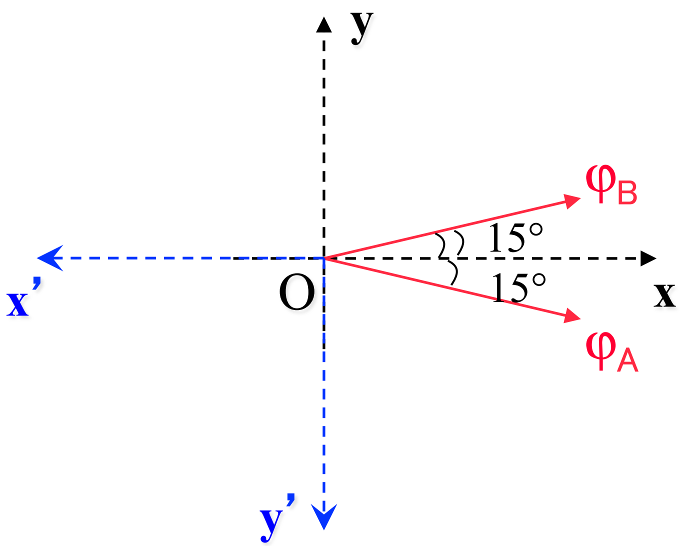
$$AS = \sigma_\phi = \sqrt{\int_0^{2\pi} \left(\phi - \bar{\phi}\right)^2 p_\phi(\phi) \, d\phi}$$

In the discrete case we have:

$$AS = \sqrt{\sum_{i=1}^{N} p_i \cdot \left(\phi_i - \bar{\phi}\right)^2}$$

Azimuth Spread problem

Example: 2 paths with equal power and different directions of arrival, f_A and f_B



$$p_\phi(\phi) = \frac{1}{2}\delta(\phi - \phi_A) + \frac{1}{2}\delta(\phi - \phi_B)$$

2 different reference systems: Oxy and Ox' y'

The azimuth angles are measured from the x and x' axis counterclockwise

Adopting the reference system Oxy, we have:

$\phi_A = 345°$ $\phi_B = 15°$

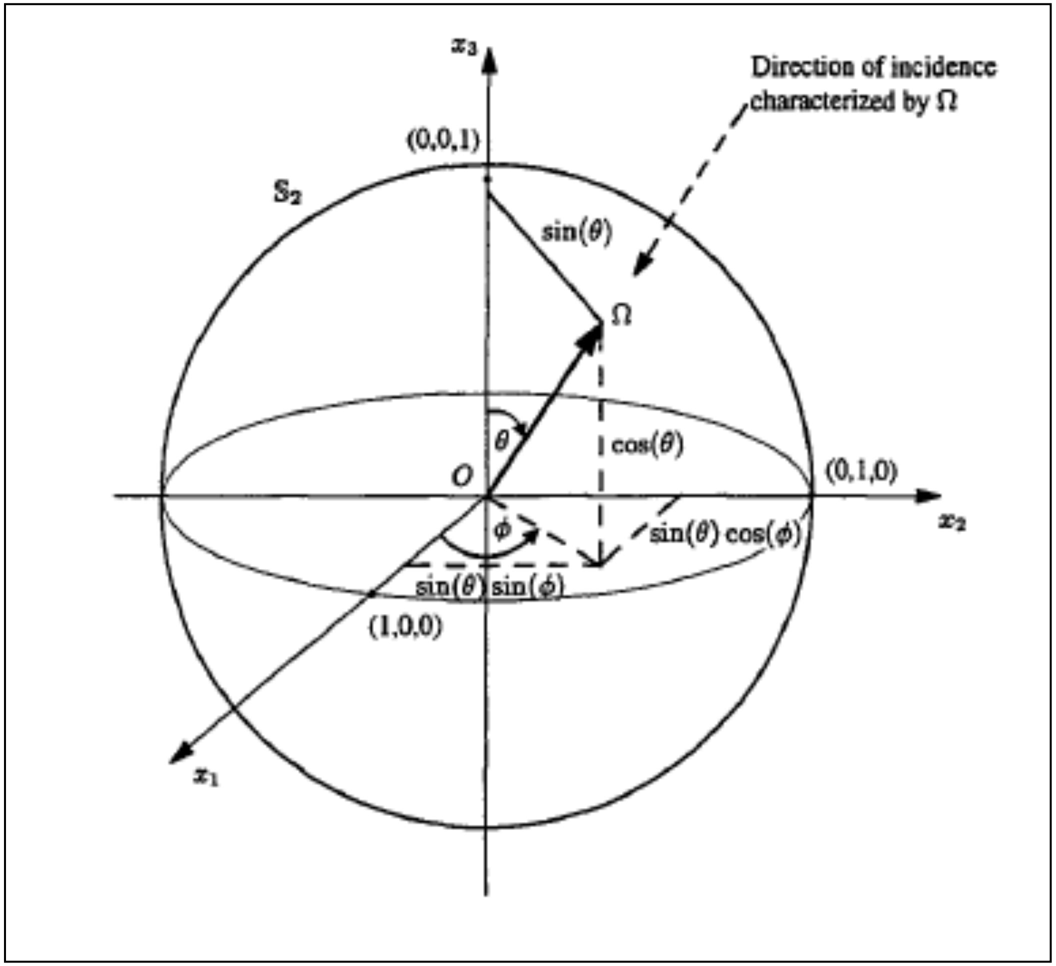
$\Rightarrow$ $\bar{\phi} = 180°$ $\sigma_\phi = 165°$

Adopting the reference system Ox' y', we have: $\phi_A = 165°$, $\phi_B = 195°$

$\Rightarrow$ $\bar{\phi} = 180°$ $\sigma_\phi = 15°$!!!

The reference system yielding the minimum spread should always be adopted

3D Angle-spread



Each direction can be represented by a unit vector $\vec{\Omega} = \vec{\Omega}(\theta,\phi)$. The initial point of $\vec{\Omega}$ is anchored at the reference location O, while its tip is located on a sphere of unit radius centered on O (see figure)

$$\vec{\Omega} = \vec{\Omega}(\theta,\phi) = \left[\cos(\phi)\sin(\theta), \sin(\phi)\sin(\theta), \cos(\theta)\right]^T$$

3D Angle-spread (II)

Mean Direction Of Arrival (DOA):

$$\left\langle \vec{\Omega} \right\rangle = \int_{4\pi} \vec{\Omega}\, p_{\Omega}\left(\vec{\Omega}\right) d\Omega$$

$p_{\Omega}\left(\vec{\Omega}\right)$ 3D power-angle profile

3D angle spread[*]:

$$AS^{3D} = \sigma_{\vec{\Omega}} = \sqrt{\int_{4\pi} \left|\vec{\Omega} - \left\langle \vec{\Omega} \right\rangle\right|^2 p_{\Omega}\left(\vec{\Omega}\right) d\Omega} = \sqrt{\left\langle \left|\vec{\Omega}\right|^2 \right\rangle - \left|\left\langle \vec{\Omega} \right\rangle\right|^2} = \sqrt{1 - \left|\left\langle \vec{\Omega} \right\rangle\right|^2}$$

(the last equality results from: $\left|\vec{\Omega}\right| = 1$)

In the discrete case the definitions above become:

$$\left\langle \vec{\Omega} \right\rangle = \sum_{k=1}^{N} p_k \vec{\Omega}_k$$

$$\sigma_{\vec{\Omega}} = \sqrt{\sum_{k=1}^{N} \left|\vec{\Omega} - \left\langle \vec{\Omega} \right\rangle\right|^2 p_k} = \sqrt{1 - \left|\left\langle \vec{\Omega} \right\rangle\right|^2}$$

3D Angle-spread (III)

- $\sigma_{\vec{\Omega}}$ does not depend on the choice of the reference system in the RX location

- $\sigma_{\vec{\Omega}}$ provides a 3D description of the angle dispersion of the channel.

- Notice that, in general, results: $\sigma_{\vec{\Omega}} \in [0\ ,\ 1]$

  Therefore it has the meaning of percentage of the whole solid angle

A completely similar formulation holds for the angle of departure

Extension to the space domain (1/2)

Let's consider the basic low-pass channel transfer function:

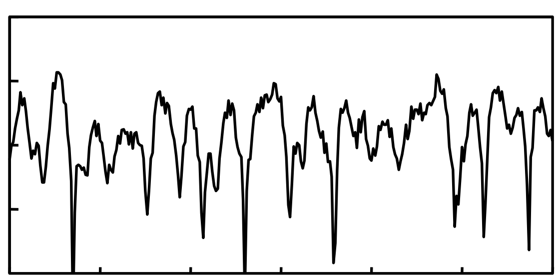
$$H(t,f)=\sum_i \rho_i e^{j\left\{2\pi f_i t-2\pi\left(f+f_0\right)t_i+\vartheta_i\right\}}$$

It is useful to consider the space domain, for example in terms of the position of the receiver **r**=(x,y,z). It is known that the channel changes over small-scale **r** changes (e.g: fast fading). Therefore we can assume:

$$H(t,f,\mathbf{r})=\sum_i \rho_i f_i\left(\mathbf{r}\right) e^{j\left\{2\pi f_i t-2\pi\left(f+f_0\right)t_i+\vartheta_i\right\}}$$

It is enough to note here that r-dependecy must be of the e-kind since angle dependency is of the δ-kind. We can have for example (typical Rayleigh fading pattern):

$$|H(\mathbf{r})| \propto \left|\sum_i f_i\left(\mathbf{r}\right)\right|$$



space, r

$$H(\mathbf{r})=H(0,0,\mathbf{r})$$

Extension to the space domain (2/2)

The space-dependance is Fourier-related to the angle dependance.
It can be shown for example that H(x,y) on a plane is a 2-D Fourier transform of the corresponding H(θ,φ) (Fourier's optics, not covered here).

Therefore the space domain can be defined e-kind while the angle domain can be defined δ-kind

Examples:

1 path case: $p_\phi(\phi) = \delta(\phi - \phi_0) \quad \Rightarrow \quad f(\mathbf{r}) = \text{constant}$

uniform 2D case: $p_\phi(\phi) = \dfrac{1}{2\pi} \quad \Rightarrow \quad \left|\sum_i f_i(\mathbf{r})\right| \simeq$ random Rayleigh fading (no dim.)

Jakes Model

*Space dependence of the envelope (es: |H(**r**)|) is what is called elsewhere fast fading.*
If fading is flat in frequency, absolute time, can still be "selective" in space.

Envelope correlations (1/5)

It is useful to define transfer function's envelope-correlations. Considering the module of the generic transfer function |M(z)| in a e-kind domain z, the domain span Δz and the average value $\left|\bar{M}\right|_{\Delta z}$ over Δz we have:

"z-wise" correlation (envelope correlation)

$$R_z(\delta) = \frac{\int_{\Delta z}\left[\left|M(z)\right| - \left|\bar{M}\right|_{\Delta z}\right]\left[\left|M(z+\delta)\right| - \left|\bar{M}\right|_{\Delta z}\right]dz}{\int_{\Delta z}\left[\left|M(z)\right| - \left|\bar{M}\right|_{\Delta z}\right]^2 dz}; \quad R_z(0) = 1, \; -1 < R_z(\delta) \le 1$$

$$\lim_{\delta\to\infty}\left\{R_z(\delta)\right\} = 0$$

Especially absolute time, frequency and space correlations are useful. The last one is fundamental for diversity techniques and MIMO.

Envelope correlations (2/5)

<u>Ex: frequency correlation</u>

$$R_f(w) = \frac{\int_{\Delta f} \left[\left| H(f) \right| - \left| \bar{H} \right|_{\Delta f} \right] \left[\left| H(f+w) \right| - \left| \bar{H} \right|_{\Delta f} \right] df}{\int_{\Delta f} \left[\left| H(f) \right| - \left| \bar{H} \right|_{\Delta f} \right]^2 df}$$

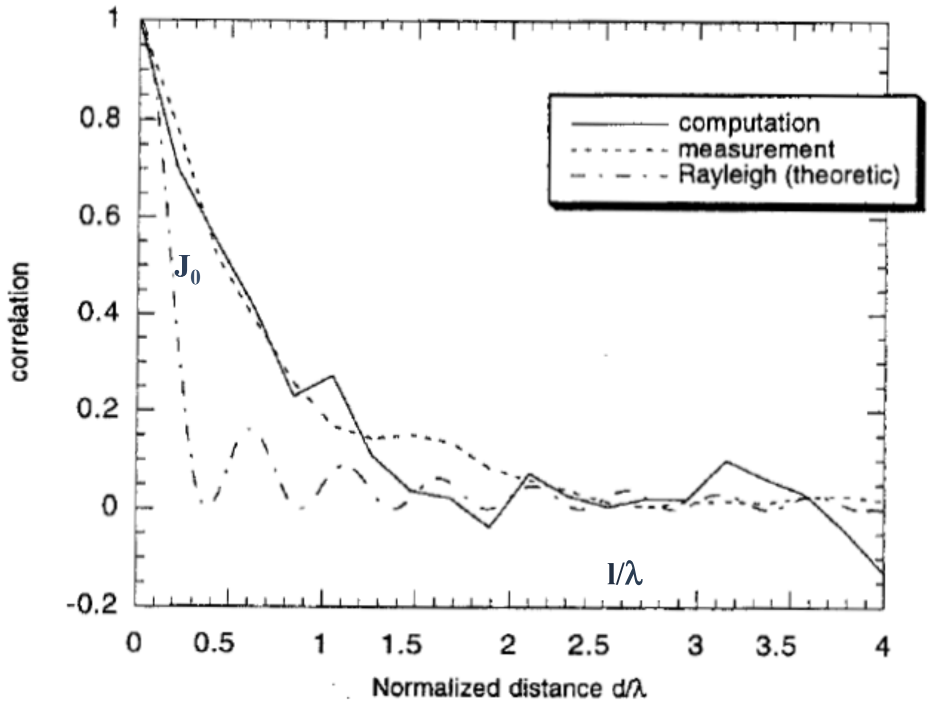
<u>Space correlation</u> (along the x direction)

$$R_x(l) = \frac{\int_{\Delta x} \left[\left| H(x) \right| - \left| \bar{H} \right|_{\Delta x} \right] \left[\left| H(x+l) \right| - \left| \bar{H} \right|_{\Delta x} \right] dx}{\int_{\Delta x} \left[\left| H(x) \right| - \left| \bar{H} \right|_{\Delta x} \right]^2 dx}$$

Envelope correlations (3/5)

Ex. space correlation in a Rayleigh environment, i.e. with uniform 2D power-angle distribution with $p_\varphi(\varphi)=1/2\pi$ is:

$$R_x(l) = J_0\left(\frac{2\pi l}{\lambda}\right)$$



With J_0 the zero order Bessel's function of the first kind. This means that the signal received from two Rx's $\lambda/2$ apart is nearly uncorrelated (see figure), and this can be useful to decrease fast fading effects

Envelope correlations (4/5)

Frequency correlation and time correlation allow a rigorous definition of coherence bandwidth and coherence time.
Given a reference, residual frequency correlation "a", then coherence bandwidth is:

$$B_C^{(a)} = \bar{w} \;\; with \;\; R_f(w) \le a \;\; \text{for} \;\; \mathrm{w} \ge \bar{w}$$

Similarly, given a reference, residual time correlation "a", then coherence time is :

$$T_C^{(a)} = \bar{t} \;\; with \;\; R_t(t) \le a \;\; \text{for t} \ge \bar{t}$$

Coherence distance L_c can also be defined in the following way:

$$L_C^{(a)} = \bar{l} \;\; with \;\; R_x(l) \le a \;\; \text{for } l \ge \bar{l}$$

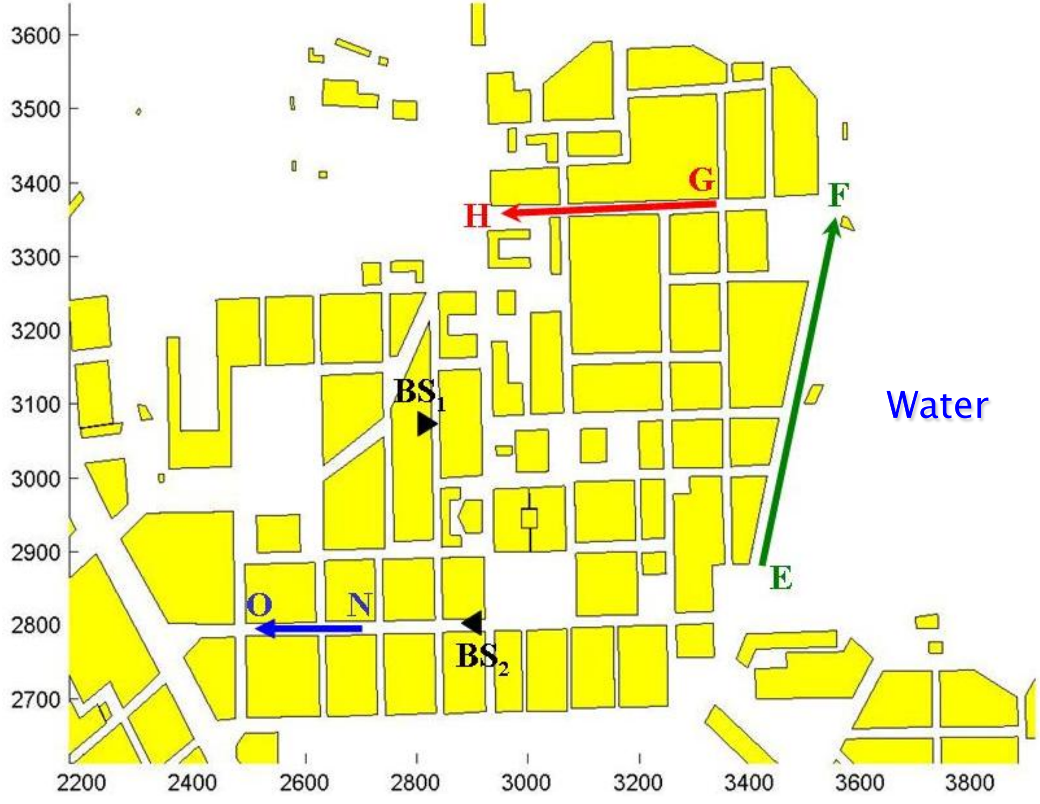
Envelope correlations (5/5)

The higher the coherence distance L_c the lower the angle spread.
All considered we have:

$$B_c^{0.1} \simeq \frac{1}{\sigma_\xi} \qquad T_c^{0.1} \simeq \frac{1}{\sigma_\nu} \qquad L_c^{0.1} \propto \frac{1}{\sigma_{\vec{\Omega}}} \approx \frac{\lambda}{4}\frac{1}{\sigma_{\vec{\Omega}}}$$

Examples: ray tracing simulations

The scenario – central Helsinki

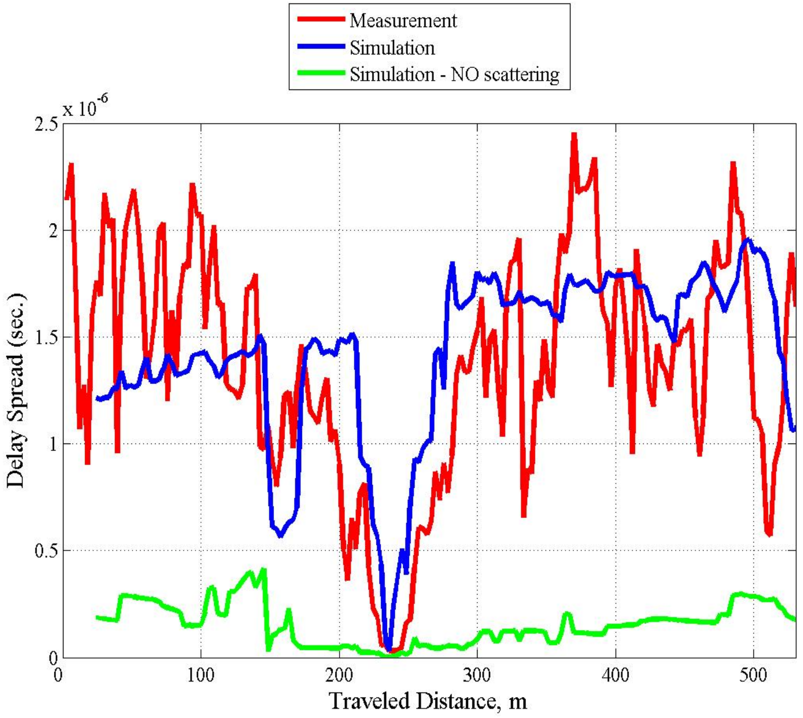


- Dense urban area
- TX: 3m above rooftop
- Rx on a trolley

  h ≈ 1.5m

- RT simulation with only 3 events

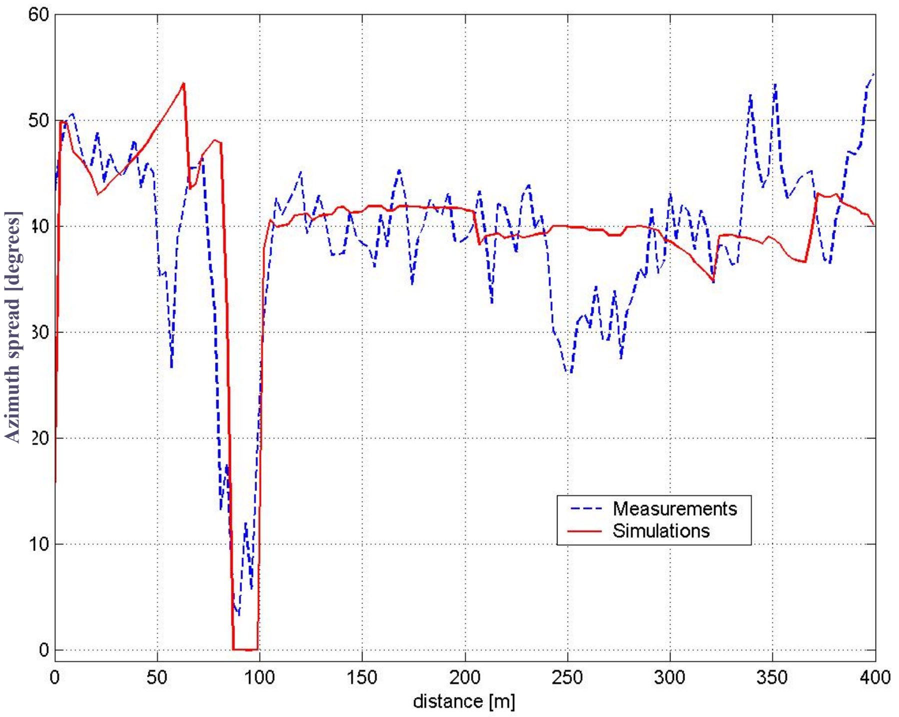
Route EF (1/3) [*]



[*] measurements by Helsinki University of Technology

Route EF (2/3)

3600
3500
3400
3300
3200
3100
3000
2900
2800
2700
2200
2400
2600
2800
3000
3200
3400
3600
3800
G
H
F
E
BS1
BS2
O
N
X

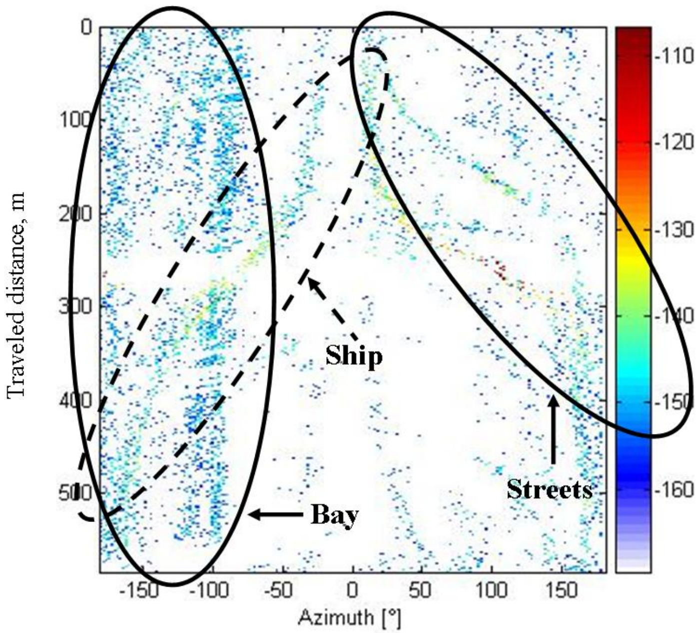


Azimuth spread [degrees]
distance [m]
Measurements
Simulations
0
10
20
30
40
50
60
0
50
100
150
200
250
300
350
400

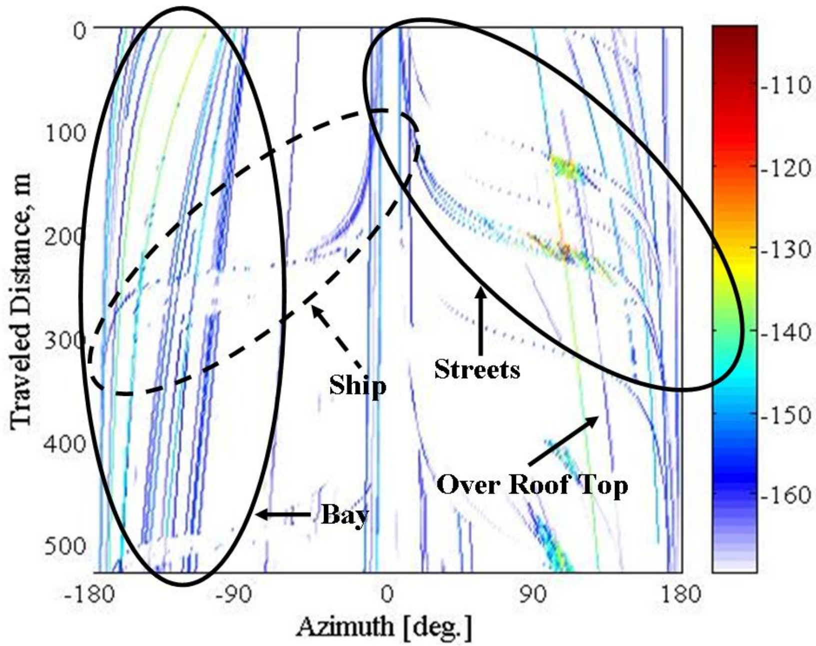
Route EF (3/3)

azimuth-distance plot

measured [*]



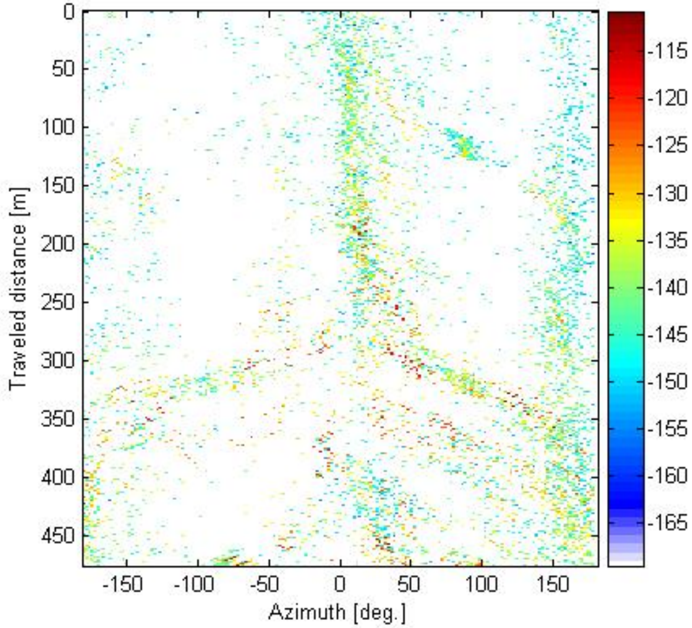
RT simulated



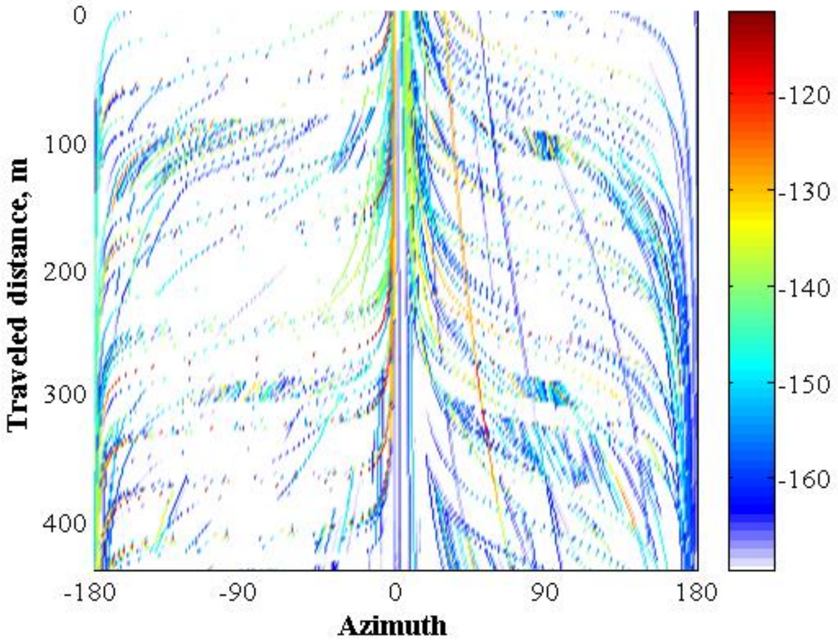
Route GH

azimuth-distance plot

measured [*]



RT simulated



Multidimensional prediction by ray tracing
Example

Helsinki route GH: azimuth-delay animation

(RT simulation only)