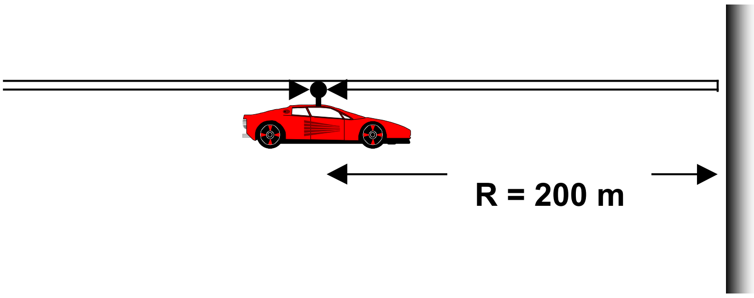
Exercise E1

A mobile station receives from a transmitter 2 waves: a direct wave and a reflected wave coming from opposite directions. (see figure).



The mobile station moves toward the reflecting wall with a constant speed of 6 m/sec. Carrier frequency is 1.8 GHz.

Assuming power attenuation due to reflection equal to 6 dBs and that excess attenuation due to path length difference be negligible, determine:

1. The deterministic RMS *Delay spread* when the receiver is at 200 m from the wall.
2. The deterministic RMS *Doppler spread* in the same conditions as before.
3. Considering now a successive time instant when the mobile is closer to the wall. Will the RMS delay spread increase or decrease?

SOLUTION

Be P_0 the direct ray power and P_r the reflected ray power. It is:

$P_r = P_0 / A$

Attenuation A according to the assumptions, is only due to reflection, therefore:

$$A = 10^{6/10} \approx 4 \qquad \Rightarrow \qquad P_r = \frac{P_0}{4} \qquad \Rightarrow \qquad P_{tot} = \frac{5}{4} P_0$$

Then, assuming the direct-ray relative delay equal to zero, we have:

$$T_{M0} = \xi_0 \cdot \frac{P_0}{P_{tot}} + \xi_r \cdot \frac{P_r}{P_{tot}} = 0 \cdot \frac{4}{5} + \frac{400}{3 \cdot 10^8} \cdot \frac{1}{5} = 0.26\ \mu\text{sec}$$

$$DS = \sqrt{T_{M0}^2 \cdot \frac{P_0}{P_{tot}} + \left(\xi_r - T_{M0}\right)^2 \cdot \frac{P_r}{P_{tot}}} = \sqrt{\left(0.26 \cdot 10^{-6}\right)^2 \cdot \frac{4}{5} + \left(1.33 \cdot 10^{-6} - 0.26 \cdot 10^{-6}\right)^2 \cdot \frac{1}{5}} =$$

$$= \sqrt{0.05408 \cdot 10^{-12} + 0.229 \cdot 10^{-12}} = 0.53 \cdot 10^{-6}\ \text{sec}$$

Doppler spread calculation is very similar.

By definition, the Doppler shift for a generic wave is :

$$f_d = -f_0 \cdot \frac{\vec{v} \cdot \hat{i}}{c}$$

where f_0 is the nominal carrier frequency, $\vec{v}$ mobile speed vector and $\hat{i}$ the direction of arrival versor.

Therefore:

$$f_{d0} = -f_0 \cdot \frac{|\vec{v}|}{c} = -1.8 \cdot 10^9 \cdot \frac{6}{3 \cdot 10^8} = -36\, Hz$$

$$f_{dr} = +f_0 \cdot \frac{|\vec{v}|}{c} = +1.8 \cdot 10^9 \cdot \frac{6}{3 \cdot 10^8} = +36\, Hz$$

$$W_0 = f_{d_0} \cdot \frac{P_0}{P_{tot}} + f_{d_r} \cdot \frac{P_r}{P_{tot}} = -36 \cdot \frac{4}{5} + 36 \cdot \frac{1}{5} = -\frac{108}{5} = -21.6 \text{ Hz}$$

$$W = \sqrt{\left(f_{d0} - W_0\right)^2 \cdot \frac{P_0}{P_{tot}} + \left(f_{dr} - W_0\right)^2 \cdot \frac{P_r}{P_{tot}}} = \sqrt{\left(-36 + 21.6\right)^2 \cdot \frac{4}{5} + \left(36 + 21.6\right)^2 \cdot \frac{1}{5}} =$$

$$= 28.8 \text{ Hz}$$

If the mobile comes closer to the wall then while powers do not change (assumption) the relative delay ξ_r decreases, therefore, the DS value decreases. Of course DS would be equal to zero on the wall due to the null relative delay ξ_r .

On the contrary, there is no Doppler spread change since Doppler shifts do not change.