Exercise A2

Problem 1

Let's consider a 1800 MHz public mobile radio system operating in a microcellular environment. The base station is at a height of 3 m above the ground and the mobile at 1 m. The antenna gains are 5 and 3 dBi's for the BS and the mobile antenna, respectively.

It is required to determine the minimum transmitted power in order for the BS to serve a mobile at a distance of 1000 m in LOS. Receiver sensitivity is -104 dBm. Ground reflection must be taken into account.

Problem 2

Now the mobile turns into a perpendicular street and and goes another 100 meters down the street. The same minimum Tx power as in problem 1 must be determined, this time neglecting ground reflection, assuming the same height for both terminals and using GTD diffraction coefficients. Both the BS and the mobile are at 10 m from the side of the street. Polarization is horizontal.

Solution

Problem 1

Friis's equation is:

$$Pt(dBm) = Pr(dBm) - Gt - Gr + 20*\log(4\pi d/\lambda)$$

If only the direct ray is considered we have

$$Pt'(dBm) = -104 -5 -3 +97.6= -14.4 \text{ dBm}$$

If also ground reflection is considered we have an additional path gain of

$$20*Log\left\{\frac{|E|}{|E_0|}\right\} = 20*Log\left\{2\left|\sin\left(\frac{2\pi}{\lambda}\frac{h_1 h_2}{d}\right)\right|\right\} = -13.15dB$$

Since link distance is much greater than breakpoint distance (about 72 m here) also the simplified formula could be used:

$$20*Log\left\{\frac{|E|}{|E_0|}\right\} = 20*Log\left\{\frac{4\pi}{\lambda}\frac{h_1 h_2}{d}\right\} = -12.91dB$$

Using the first, more accurate result we get:

$$Pt(dBm) = Pt' + 13.15 = -0.9 \text{ dBm}$$

Problem 2

Adopting a similar approach as before, if diffraction would cause no attenuation we'd have a total path of

$$s'+s = \sqrt{1000^2+10^2} + \sqrt{100^2+10^2} \simeq 1000+100.5 = 1100.5m$$

Without diffraction we would therefore get:

$$Pt'(dBm) = -104 -5 -3 +98.4= -13.6 \text{ dBm}$$

Now let's derive the additional path gain with diffraction

$$PG = 20 * Log\left\{\frac{|E|}{|E_0|}\right\}$$

The field of the unfolded ray without diffraction would be

$$\mathbf{E}_0 = j\sqrt{\frac{\eta PG_{Tx}}{2\pi}}\frac{e^{-j\beta r}}{r}\mathbf{i}_x = j\sqrt{60EIRP}\frac{e^{-j\beta r}}{r}\mathbf{i}_x$$

Thus:

$$|\mathbf{E}_0| = \frac{\sqrt{60EIRP}}{s+s'} \quad (1)$$

Where s+s' is the total unfolded ray length. Now, if P_d is the diffraction point on the edge, for the diffracted field we have:

$$|E| = |E(P_d)| \cdot |D| \cdot A(s,s') = \frac{\sqrt{60EIRP}}{s'}|D_H| \cdot \sqrt{\frac{s'}{s(s+s')}}$$

Therefore (see also the appendix)

$$PG = 20 * Log\left\{\frac{|E|}{|E_0|}\right\} = 20 * Log\left\{|D_H| \cdot \frac{s+s'}{s'}\sqrt{\frac{s'}{s(s+s')}}\right\} =$$

$$20 * Log\left\{|D_H| \cdot \sqrt{\frac{s+s'}{s \cdot s'}}\right\} \simeq -36.06\,dB$$

And transmitted power must be

Pt = Pt'+36.06= -13.6+36.06≈ 22.5 dBm≈ 178 mW

A more straightforward method is to compute the field at the diffraction point using a formula similar to (1), then compute the diffracted field in the destination point with the usual GTD formula, and finally derive the received power (as a function of the Tx power) by multiplying the destination incident power density by the effective area of the Rx antenna.

Adopting this approach we have:

$$|E| = |E(P_d)| \cdot |D| \cdot A(s,s') = \frac{\sqrt{60EIRP}}{s'}|D_H| \cdot \sqrt{\frac{s'}{s(s+s')}}$$

where

$$A(s,s') = \sqrt{\frac{s'}{s(s+s')}} = 0.095 \quad \text{and} \quad |D_H| = 0.15$$

therefore:

$$P_R = \frac{|E|^2}{2\eta}G_R\frac{\lambda^2}{4\pi} = -104 \text{ [dBm]} = 39.8\text{x}10^{-15} \text{ [W]}$$

Thus :

$$\left|E\right|^2 = P_R \frac{2\eta \cdot 4\pi}{G_R \cdot \lambda^2} = P_R \frac{8 \cdot 120\pi^2}{G_R \cdot \lambda^2}$$

$$\Downarrow$$

$$\frac{\sqrt{60 EIRP}}{s'} \left|D_H\right| \cdot A = \sqrt{P_R \frac{8 \cdot 120\pi^2}{G_R \cdot \lambda^2}}$$

$$\Downarrow$$

$$EIRP = P_R \frac{8 \cdot 2\pi^2}{G_R \cdot \lambda^2} \frac{s'^2}{\left|D_H\right|^2 \cdot A^2}$$

$$\Downarrow$$

$$P_T = P_R \frac{8 \cdot 2\pi^2}{G_T G_R \cdot \lambda^2} \frac{s'^2}{\left|D_H\right|^2 \cdot A^2} = 39.8x10^{-15} \frac{16\pi^2}{3.16 \cdot 2 \cdot \lambda^2} \frac{1000^2}{0.15^2 \cdot 0.095^2} = 0.177 \text{ [W]}$$

Appendix: computation of $|D_H|$

The GTD expression is:

$$D^H(\phi, \phi', n) = \frac{-e^{-j\pi/4} \cdot \sin\left(\pi/n\right)}{n\sqrt{2\pi\beta}} \cdot \left[\frac{1}{\cos\left(\pi/n\right) - \cos\left(\xi^-/n\right)} + \frac{1}{\cos\left(\pi/n\right) - \cos\left(\xi^+/n\right)} \right] \quad (1)$$

In this case:

n =1.5

ϕ=1.5π-atan(10/100)=4.61 Rad

ϕ'=atan(10/1000)=0.01

ζ^-=ϕ- ϕ'=4.6

ζ^+=ϕ+ϕ'=4.62

The square parenthesis in 1) gives therefore

$$\left[\frac{1}{\cos\left(\pi/1.5\right) - \cos\left(4.60/1.5\right)} + \frac{1}{\cos\left(\pi/1.5\right) - \cos\left(4.62/1.5\right)} \right] = [2.01 + 2.008] = 4.018$$

While the preceding factor gives:

$$\left| \frac{-e^{-j\pi/4} \cdot \sin\left(\pi/n\right)}{n\sqrt{2\pi\beta}} \right| = \frac{\sin\left(\pi/1.5\right)\sqrt{\lambda}}{1.5 \cdot 2\pi} = 0.037$$

Thus:

$$\left|D^H\right| = 0.037 \cdot 4.018 = 0.15$$