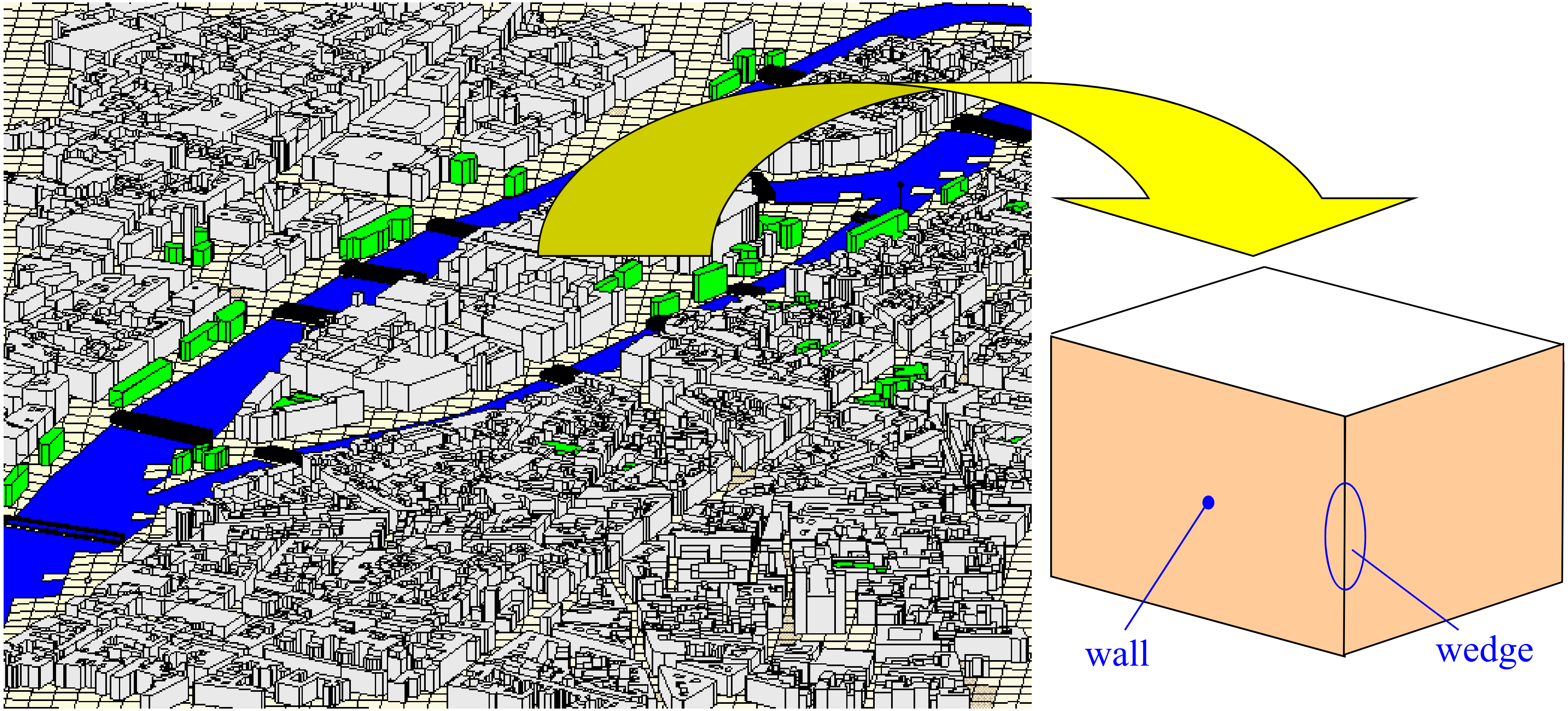
# A - TEORIA DELLA PROPAGAZIONE RADIO IN AMBIENTE REALE

- Effetto di gas atmosferici e idrometeore
  - Attenuazione supplementare da gas atmosferici
  - Attenuazione supplementare da pioggia
  - Propagazione ionosferica, troposcatter
- Propagazione in mezzi con disomogenità distribuita – Propagazione troposferica
  - Cenni di ottica geometrica in mezzi con n debolmente variabile.
  - Propagazione in mezzi a stratificazione piana e sferica. Propagazione troposferica, orizzonte radio e rettificazione del suolo/raggio
- Propagazione in mezzi con disomogenità concentrate – Propagazione in presenza di ostacoli
  - Riflessione del suolo, diffrazione da knife-edge, ellissoide di Fresnel
  - Metodi per il calcolo della diffrazione da ostacoli
  - Teoria geometrica della propagazione: trasmissione attraverso uno strato, diffrazione da spigolo. Propagazione multicammino.

# Geometrical theory of propagation (I)

It is useful when propagation takes place in a region with concentrated obstacles. Obstacles are here represented as plane walls and rectilinear wedges (*canonical obstacles*)



# Geometrical theory of propagation (II)

- Geometrical theory of propagation is an extension of Geometrical Optics, (GO) and is not limited to optical frequencies $\left(\lambda \to 0 \ \text{ so that } \ \Delta n \to 0 \ \text{ over } \ \lambda\right)$
- Like GO, it corresponds to an asymptotic, high-frequency approximation of basic electromagnetic theory, and is based on the *ray concept*
- Since GO does not account for diffraction, then diffraction is introduced through an extension called Geometrical Theory of Diffraction (GTD)
- The combination of GO and GTD, applied to radio wave propagation may be called Geometrical Theory of Propagation (GTP) and is the base of deterministic, ray propagation models (ray-tracing etc.)

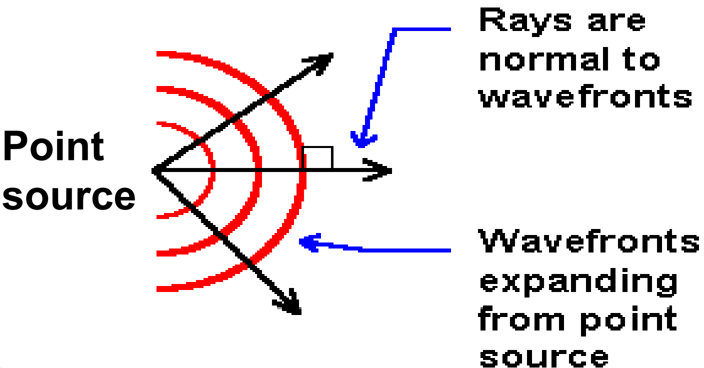
# Recall: waves and rays

- <u>Wavefront</u>: surface were the field has the same phase (varies “in phase”)

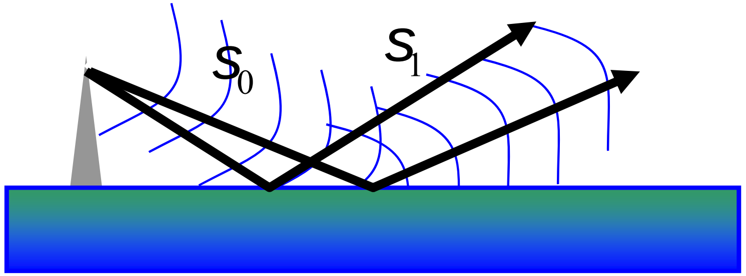
- <u>Ray</u>: given a propagating wave, every curve that is everywhere perpendicular to the wavefront is called ray. A ray is *the path* of a wave. There is a mutual identification btw wave and ray

- In presence of <u>concentrated obstacles</u> rays are <u>piecewise-rectilinear and wavefronts can be of various kinds</u> (see further on)

*Ex.1 Sperical wave and* ***rectilinear*** *rays*

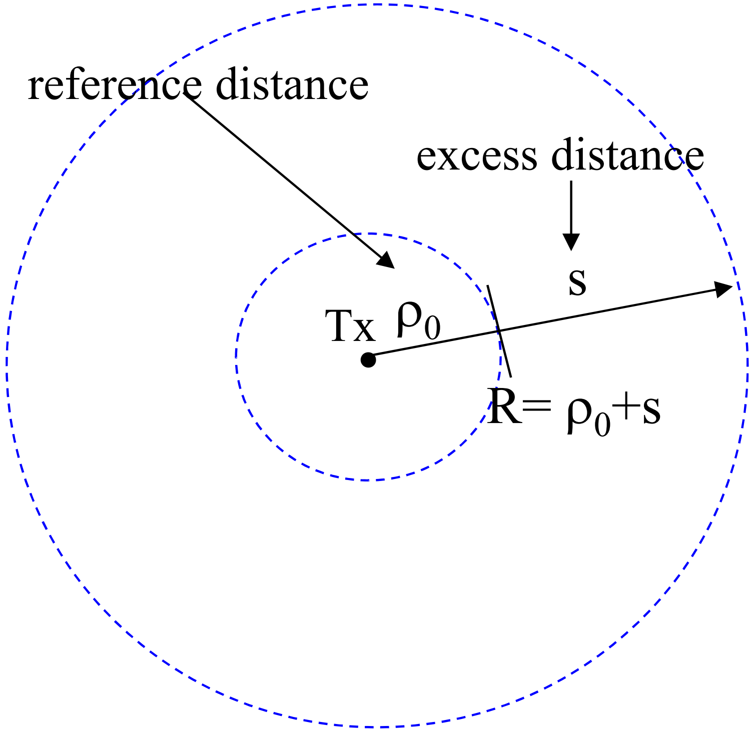


*Ex.2 reflected spherical wave and* ***piece-wise rectilinear*** *rays*



# The spherical wave

In free space the reference wave is the spherical wave



field:

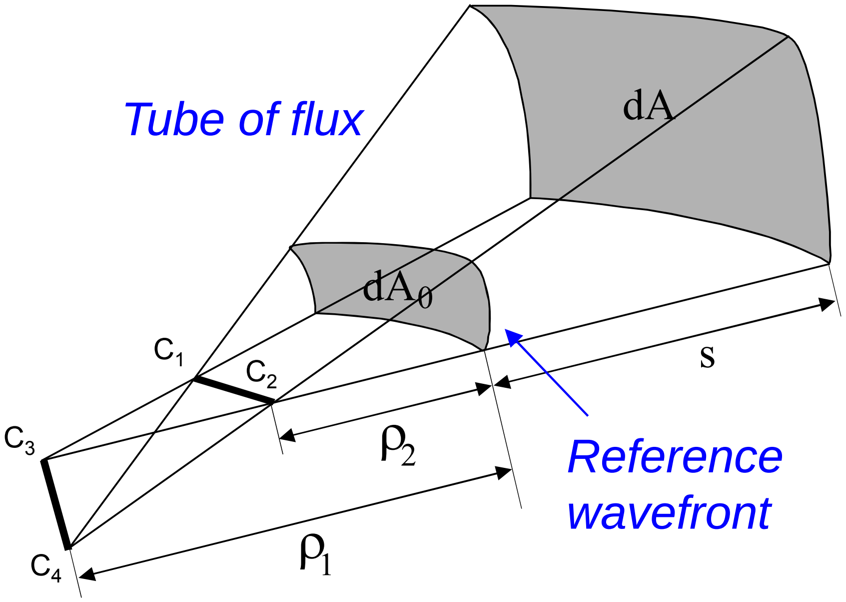
$$\mathbf{E}(R)=\mathbf{E}_0\frac{e^{-j\beta R}}{R}=\mathbf{E}_0\frac{e^{-j\beta\rho_0}}{\rho_0}\;\frac{\rho_0}{R}e^{-j\beta(R-\rho_0)}=$$

$$\mathbf{E}(\rho_0)\frac{\rho_0}{R}e^{-j\beta(R-\rho_0)}=\mathbf{E}(\rho_0)\frac{\rho_0}{\rho_0+s}e^{-j\beta(s)}$$

power density:

$$p(R)=p(\rho_0)\left(\frac{\rho_0}{\rho_0+s}\right)^2$$

# The astigmatic wave: divergence factor



If the mean is homogeneous (➔ rectilinear rays) [2] the generic wave's divergence factor is:

$$\boxed{A(\rho_1,\rho_2,s)=\sqrt{\frac{\rho_1\cdot\rho_2}{(\rho_1+s)\cdot(\rho_2+s)}}\left(=\sqrt{\frac{dA_0}{dA}}\right)}$$

- ***A* : *Divergence or Spreading factor***
- $\rho_1$, $\rho_2$ : curvature radii
- $\overline{C_1C_2}$ , $\overline{C_3C_4}$ : wave caustics

There are 3 main reference cases:

- Spherical wave: $\rho_1 = \rho_2 = \rho_0$ ➔ $A = \dfrac{\rho_0}{\rho_0 + s}$
- Cylindrical wave: $\rho_1 = 0$, $\rho_2 = \rho_0$ ➔ $A = \sqrt{\dfrac{\rho_0}{\rho_0 + s}}$
- Plane wave: $\rho_1 = \rho_2 = \infty$ ➔ $A = 1$

notice that, for power conservation:

$$A=\sqrt{\frac{dA_0}{dA}}=\sqrt{\frac{|E|^2}{|E_0|^2}}=\frac{|E|}{|E_0|}=\sqrt{\frac{1}{L}}$$

*L*: power attenuation

# The generic wave: amplitude and polarization

The divergence factor gives the field- (and thus power-) attenuation law along the ray. But since the field is a complex vector, we also have polarization.

The generic (astigmatic) wave in free space has the electric field:

$$\vec{E}(s) = \underbrace{\vec{E}(0)}_{\text{Field at reference point }(s=0)} \cdot \underbrace{\underbrace{\sqrt{\frac{\rho_1 \cdot \rho_2}{(\rho_1 + s)\cdot(\rho_2 + s)}}}_{\text{Divergence factor}} \cdot \underbrace{e^{-j\beta s}}_{\text{Phase factor}}}_{\text{Propagation factor}}$$

(Ex: spherical wave)

$$\vec{E}(s) = \vec{E}_0 \cdot \frac{\rho_0}{\rho_0 + s} e^{-j\beta s} = \hat{\mathbf{p}} K \left( \frac{e^{-j\beta\rho_0}}{\rho_0} \right) \frac{\rho_0}{\rho_0 + s} e^{-j\beta s} = \hat{\mathbf{p}} K \underbrace{\left( \frac{e^{-j\beta s_{tot}}}{s_{tot}} \right)}_{\text{Propagation factor}}$$

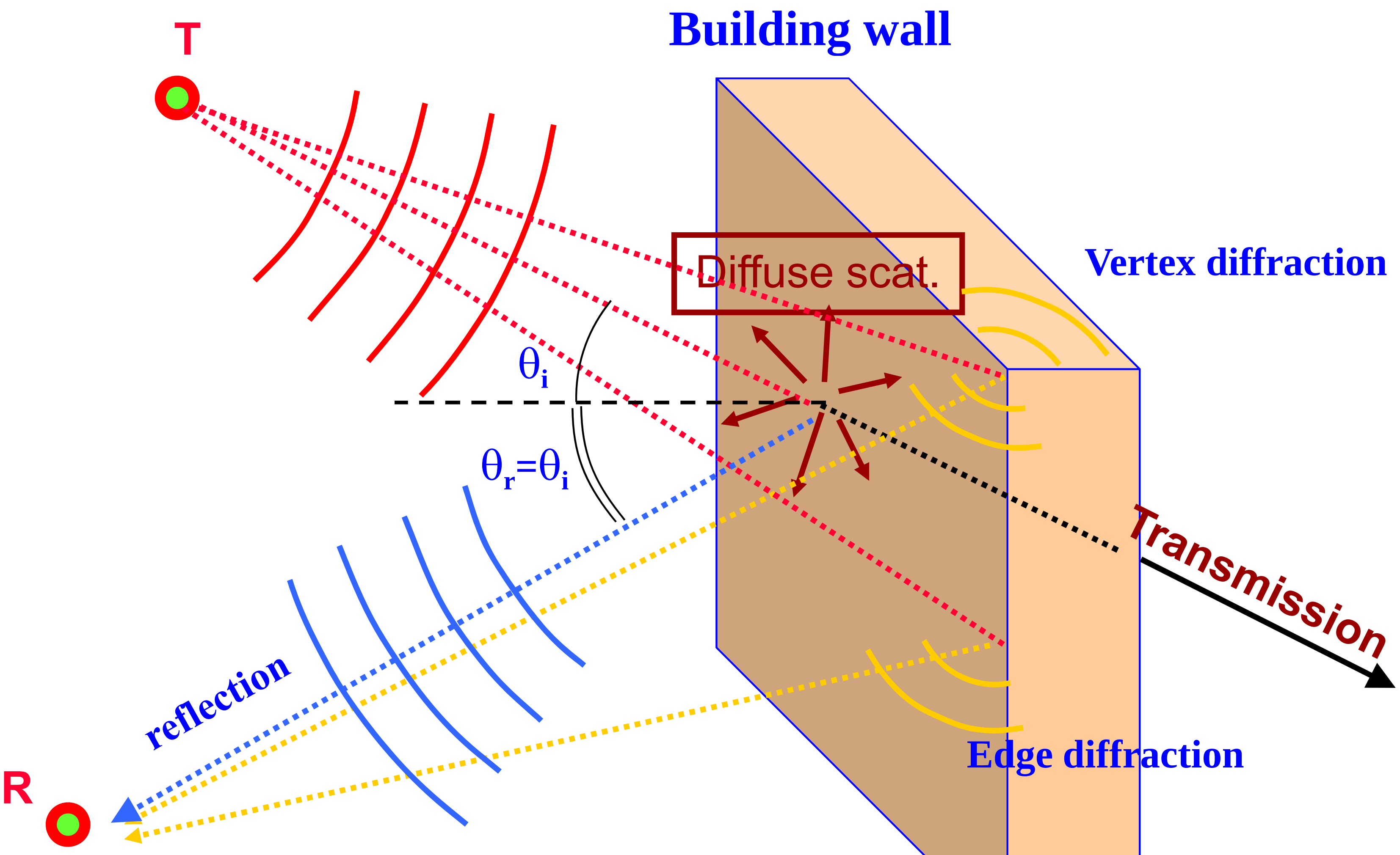
This expression gives the field amplitude *along* a ray.

The (normalized) ***polarization vector*** gives the polarization of the wave:

$$\hat{\mathbf{p}} \triangleq \frac{\vec{E}(s)}{\left|\vec{E}(s)\right|} e^{j\chi}$$

The polarization vector has the same polarization as the field but is normalized. In free space it is constant along the ray. The ***antenna polarization vector*** is the polarization vector of the field emitted by the considered antenna.

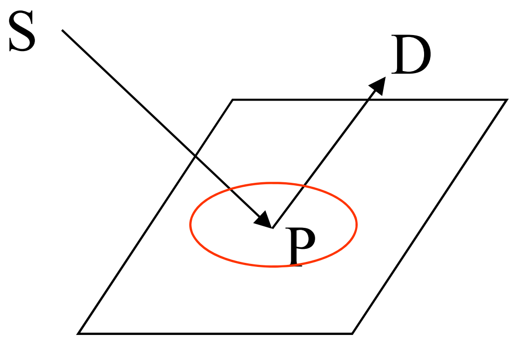
# Interaction mechanisms



# Basic principles

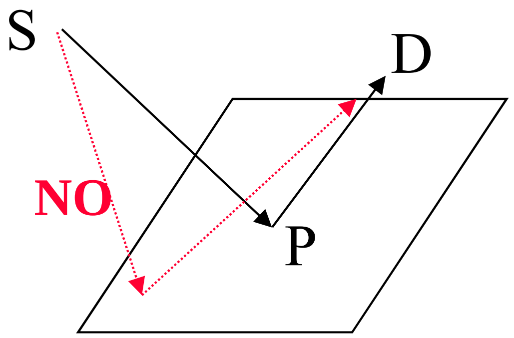
## “*Local field principle*”

- The wave can be locally assumed plane (for the interaction coefficients)
- The field associated with the reflected/transmitted/diffracted ray only depends on the electromagnetic and geometric properties of the obstacle in the vicinity of the interaction point



## “*Fermat’s principle*”

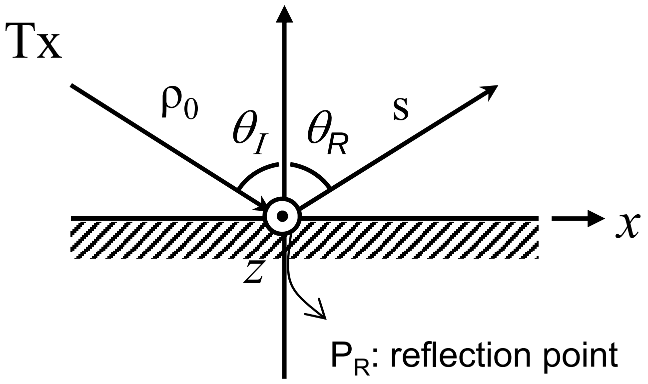
- The ray trajectory is always so as to minimize path (or optical-path …)



# The Reflected/transmitted ray

***Reflection/transmission does not change the divergence factor***

Reflected ray

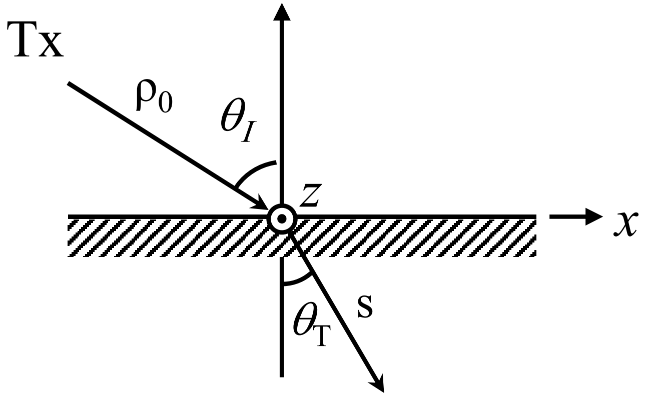


- direction: reflection law or Fermat's principle
- Field expression:

In the vicinity of $P_R$

$$\begin{bmatrix} \vec{E}_r^{TE}(s) \\ \vec{E}_r^{TM}(s) \end{bmatrix} = \begin{bmatrix} \Gamma_{TE} & 0 \\ 0 & \Gamma_{TM} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_i^{TE}(P_R) \\ \vec{E}_i^{TM}(P_R) \end{bmatrix} \cdot \frac{\rho_0}{\rho_0 + s} e^{-j\beta s}$$

Transmitted ray

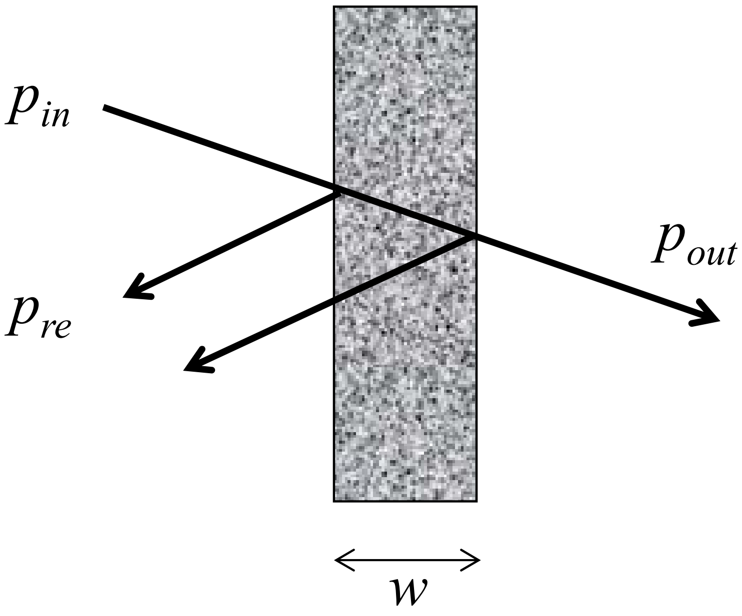


- direction: Snell's law or Fermat's principle
- Field expression:

$$\begin{bmatrix} \vec{E}_t^{TE}(s) \\ \vec{E}_t^{TM}(s) \end{bmatrix} = \begin{bmatrix} \tau_{TE} & 0 \\ 0 & \tau_{TM} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_i^{TE}(P_R) \\ \vec{E}_i^{TM}(P_R) \end{bmatrix} \cdot \frac{\rho_0}{\rho_0 + s} e^{-j\beta' s}$$

# Transmission through a wall (1/5)

* Hypotheses: - normal or quasi-normal incidence
- weakly lossy medium



$$|\Gamma| \approx |\Gamma_{TE}| = \left| \frac{\cos\theta_i - \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\left(\frac{n_2}{n_1}\right)^2 - \sin^2\theta_i}} \right| = \left| \frac{1-\sqrt{\varepsilon_r}}{1+\sqrt{\varepsilon_r}} \right|$$

$$\frac{p_{re}}{p_{in}} = \frac{|E_{re}|^2 \big/ 2\eta}{|E_{in}|^2 \big/ 2\eta} = \frac{|E_{re}|^2}{|E_{in}|^2} \approx |\Gamma|^2$$

(Source: Prof. H.L. Bertoni)

# Transmission through a wall (2/5)

In a lossy medium the wavenumber is:

$$k = \omega\sqrt{\mu_0 \varepsilon_c} = \omega\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}$$

The complex relative dielectric constant is:

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' = \frac{\varepsilon}{\varepsilon_0} - j\frac{\sigma}{\omega\varepsilon_0}$$

If the medium is weakly lossy Σ” << Σ’ .

A plane wave propagating through the lossy medium has the expression:

$$\mathbf{E} = \mathbf{E_0} e^{-jkr} = \mathbf{E_0} e^{-(\alpha + j\beta)r}$$ ; with jk=$\alpha$+j$\beta$

Thus:

$$k = \omega\sqrt{\mu_0 \varepsilon_0 \varepsilon_r} = \omega\sqrt{\mu_0 \varepsilon_0}\sqrt{\varepsilon_r' - j\varepsilon_r''} = \frac{\omega}{c}\sqrt{\varepsilon_r' - j\varepsilon_r''} =$$

$$= \frac{\omega}{c}\sqrt{\varepsilon_r'}\sqrt{1 - j\frac{\varepsilon_r''}{\varepsilon_r'}} \approx \frac{\omega}{c}\sqrt{\varepsilon_r'}\left(1 - j\frac{\varepsilon_r''}{2\varepsilon_r'}\right)$$

Where the series expansion have been truncated at first order

# Transmission through a wall (3/5)

Therefore:

$$\mathrm{Im}\left\{k\right\} \approx -\frac{\omega}{c}\sqrt{\varepsilon_r'}\left(\frac{\varepsilon_r''}{2\varepsilon_r'}\right) = -\omega\sqrt{\mu_0\varepsilon_0\varepsilon_r'}\left(\frac{\varepsilon_r''}{2\varepsilon_r'}\right) = -\frac{\pi\varepsilon_r''}{\lambda_0\sqrt{\varepsilon_r'}}$$

and :

$$jk = \alpha + j\beta \approx \frac{\omega}{c}\sqrt{\varepsilon_r'}\left(\frac{\varepsilon_r''}{2\varepsilon_r'} + j\right) \Rightarrow$$

$$\begin{cases} \alpha \approx \dfrac{\omega}{c}\sqrt{\varepsilon_r'}\left(\dfrac{\varepsilon_r''}{2\varepsilon_r'}\right) = \left|\mathrm{Im}\left\{k\right\}\right| \quad \mathrm{Im}\left\{\mathrm{k}\right\} \text{ is negative!} \\ \beta \approx \dfrac{\omega}{c}\sqrt{\varepsilon_r'} \end{cases}$$

$$\left|E(r)\right| = \left|E(0)\right| \cdot e^{-\alpha r}$$

$$p(r) = p(0) \cdot e^{-2\alpha r}$$

# Transmission through a wall (4/5)

The reflection coefficient at normal incidence for the air-medium interface is

$$\Gamma_{0m} = \frac{1-\sqrt{\varepsilon_r}}{1+\sqrt{\varepsilon_r}}$$

The reflection coefficient for the second, medium-air interface is (see the expression of the reflection coefficients for normal incidence)

$$\Gamma_{m0} = \frac{\sqrt{\varepsilon_r}-1}{1+\sqrt{\varepsilon_r}} = -\Gamma_{0m}$$

Now if we consider the first interface we have

$$\frac{p_{refl1}}{p_{inc1}} = \frac{\left|\vec{E}_{refl1}\right|^2}{\left|\vec{E}_{in1}\right|^2} = \left|\Gamma_{0m}\right|^2$$

# Transmission through a wall (5/5)

For power conservation we have:

$$p_{inc1} = p_{refl\,1} + p_{trasm1}$$

$$1 = \frac{p_{refl\,1}}{p_{inc1}} + \frac{p_{trasm1}}{p_{inc1}} = \left|\Gamma_{0m}\right|^2 + \frac{p_{trasm1}}{p_{inc1}}$$

$$\frac{p_{trasm1}}{p_{inc1}} = 1 - \left|\Gamma_{0m}\right|^2$$

Now the transmitted power at the first interface, properly multiplied by the lossy-medium attenuation factor becomes the incident power at the second interface, therefore we have

$$\frac{p_{refl\,2}}{p_{inc2}} = \left|\Gamma_{m0}\right|^2 = \left|\Gamma_{0m}\right|^2 = \left|\Gamma\right|^2$$

$$\frac{p_{transm2}}{p_{inc2}} = \frac{p_{transm2}}{P_{transm1} e^{-2\alpha w}} = \frac{p_{transm2}}{p_{inc1}\left(1-\left|\Gamma\right|^2\right) e^{-2\alpha w}} = 1 - \left|\Gamma\right|^2$$

Thus:

$$\boxed{\frac{p_{transm2}}{p_{inc1}} = \frac{p_{out}}{p_{in}} = \left(1-\left|\Gamma\right|^2\right)^2 e^{-2\alpha w} = \left(1-\left|\Gamma\right|^2\right)^2 e^{-2w\left|\operatorname{Im}\kappa\right|}}$$

# Example of Transmission Loss

Brick wall: $\varepsilon_r'=4$, $\varepsilon_r''=0.2$, $w=20$ cm

$$\frac{p_{re}}{p_{in}} \approx \left|\frac{\sqrt{4}-1}{\sqrt{4}+1}\right|^2 = \frac{1}{9} = 0.11 \quad \text{or} \quad -9.6\text{dB}$$

at 1,800 MHz ($\lambda_o=1/6$ m): $|\mathrm{Im}\,\kappa| = \dfrac{0.2\pi}{(1/6)\sqrt{4}} = 1.88$

$$\frac{p_{out}}{p_{in}} = \left(\frac{8}{9}\right)^2 e^{-2(0.2)(1.88)} = 0.37 \quad \text{or} \quad -4.3\text{dB}$$

(Source: Prof. H.L. Bertoni)

# Summary of Reflection and Transmission Loss

Theory

| Wall Type | Frequency Band | Ref. loss | Trans. Loss |
|---|---|---|---|
| Brick, exterior | 1.8 - 4 GHz | 10 dB | 10 dB |
| Concrete block, interior | 2.4 GHz | | 5 dB |
| Gypsum board, interior | 3.4 GHz | 4 dB | 2 dB |

Measured

| | | | |
|---|---|---|---|
| Exterior frame | 800 MHz | | 4 - 7 dB |
| | 5 - 6 GHz | | 9 - 18 dB |
| with metal siding | 5 GHz | | 36 dB |
| Brick, exterior | 4 - 6 GHz | 10 dB | 14 dB |
| Concrete block, interior | 2.4 / 5 GHz | | 5 / 5 - 10 dB |
| Gypsum board, interior | 2.4 / 5 GHz | | 3 / 5 dB |
| Wooden floors | 5 GHz | | 9 dB |
| Concrete floors | 900 MHz | | 13 dB |

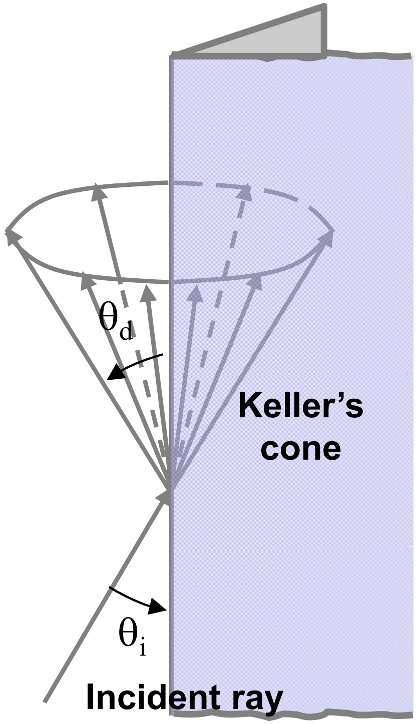
(Source: Prof. H.L. Bertoni)

# Geometrical Theory of Diffraction

The extension of GO to the category of diffracted rays was first introduced by J. B. Keller in 1961 and is based on the following assumptions[6] :

I. *A diffracted ray is generated whenever a ray impinges on an edge (or on a vertex)*

II. *For every diffracted ray the Fermat's principle holds*

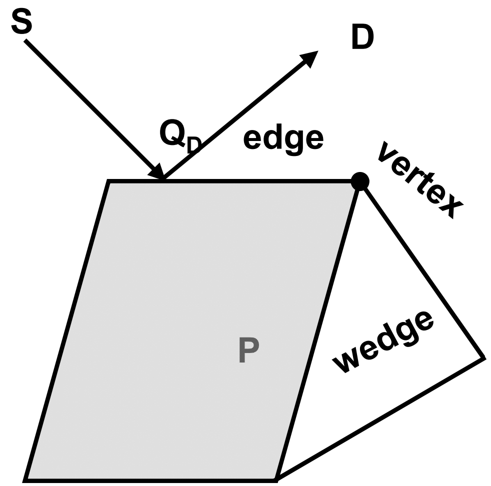


***<u>Diffraction law</u>: the angles between incident / diffracted ray and the edge satisfy "Snell's law applied to diffraction":***

$$n_i \cdot \sin\theta_i = n_d \cdot \sin\theta_d$$

➔ If the rays are in the same material then: $\theta_d=\theta_i$;
Therefore diffracted rays ouside the wedge belong to the ***Keller's cone***

# The diffracted ray (1/3)



- In urban propagation only straight edges (local field principle) are of interest. Vertex diffraction won't be treated here

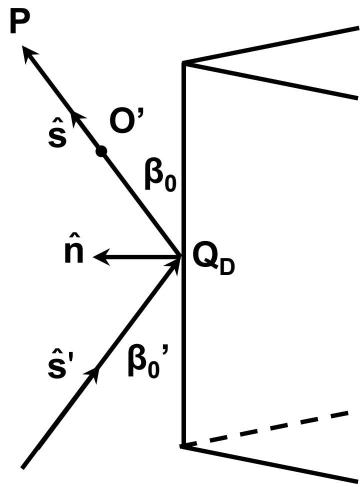
- If the impinging wave is plane (or can be approximated so for the local field principle) then the diffracted wave is cylindrical for perpendicular incidence ($\theta_d=\theta_i=\pi/2$) and conical for oblique incidence (the wavefront is a cone) [7]

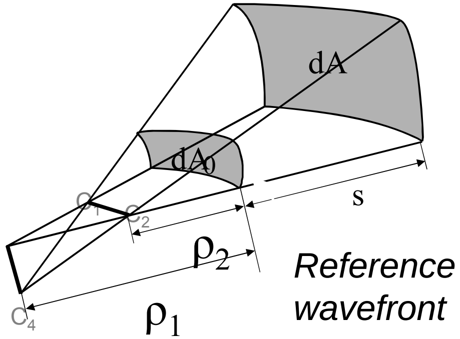
- The diffracted wave is so that one caustic coincides with the edge. **Therefore the divergence factor of the diffracted wave/ray is different from that of the incident wave/ray** (see further on)

- The diffracted ray field can be computed by solving Maxwell's equations for a plane , cylindrical or spherical wave incident on a straight conducting edge [7, 8, 9] and somehow subtracting from the solution the incident wave and the reflected wave(s).

- Then the diffracted field is expanded in a Luneberg-Kline series from which only the first term (high frequency approx.) is kept in order to derive the ***diffraction coefficients***

# The diffracted ray (2/3)





The high frequency term has the form:

$$\vec{E}^d(s) = \vec{E}^d(O') \cdot \sqrt{\frac{\rho_1^d \cdot \rho_2^d}{\left(\rho_1^d + s\right)\cdot\left(\rho_2^d + s\right)}} \cdot e^{-j\beta s}$$

$\rho_1^d$, $\rho_2^d$ = curvature radii of the diffracted wave.

<u>One caustic coincides with the edge</u>: $\rho_2^d$ corresponds to O'-$Q_D$ where O' is the reference point, origin of the coordinate s.

It is useful to choose O'=$Q_D$ ($\rho_2^d$=0 ➔ simpler expression). However for power conservation reasons $\mathbf{E}^d$(O') ➔ ∞ for O' ➔ $Q_D$

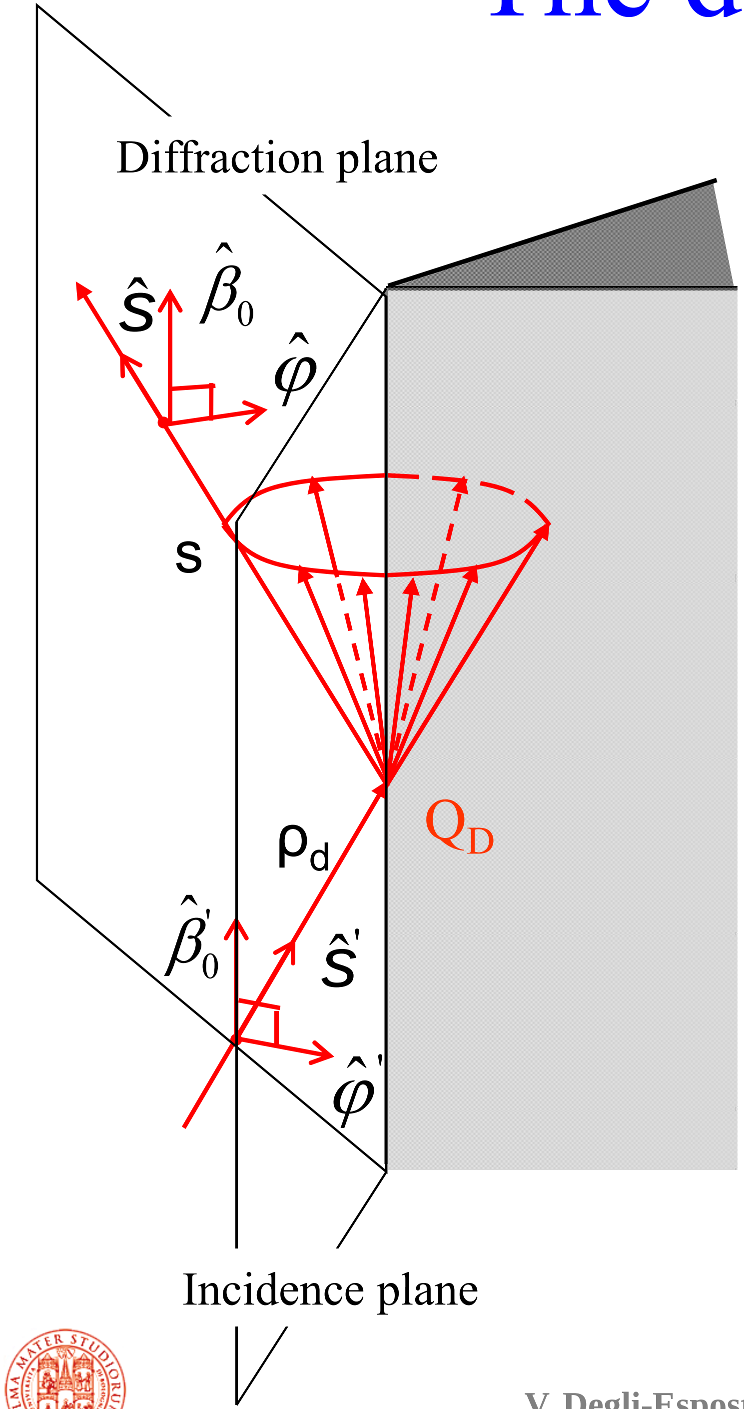
Since $\mathbf{E}^d$(s) cannot change with the reference system, therefore it must be:

$$\lim_{\substack{O'\to Q_D\\(\rho_2^d\to 0)}} \left[\vec{E}^d(O')\cdot\sqrt{\rho_2^d}\right] = \text{finite vector} \equiv \vec{E}^i(Q_D)\cdot \mathbf{D} \quad\Longrightarrow\quad \boxed{\vec{E}^d(s)=\vec{E}^i(Q_D)\cdot\mathbf{D}\cdot A\left(\rho^d,s\right)\cdot e^{-j\beta s}}$$

with: $A\left(\rho^d,s\right)=\sqrt{\dfrac{\rho^d}{\left(\rho^d+s\right)\cdot s}}$

**D** is the ***diffraction matrix,*** which contains the diffraction coefficients

# The diffracted ray (3/3)



$$\begin{bmatrix} E^d_{\beta_0} \\ E^d_{\phi} \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & D_h \end{bmatrix} \begin{bmatrix} E^i_{\beta'_0}\left(Q_D\right) \\ E^i_{\phi'}\left(Q_D\right) \end{bmatrix} \cdot A\left(s,\rho^d\right) \cdot e^{-j\beta s}$$

if the proper local reference system is adopted (see figure) then the diffraction matrix reduces to a 2x2 diagonal matrix. Each element of the matrix is the diffraction coefficient for the corresponding polarization.

ϕ-polarization is called “hard” (TE), β-polarization is called “soft” (TM)

# The divergence factor

If $\rho_2^d \rightarrow 0$ as shown, then we get :
$(\rho_1^d \rightarrow \rho^d)$

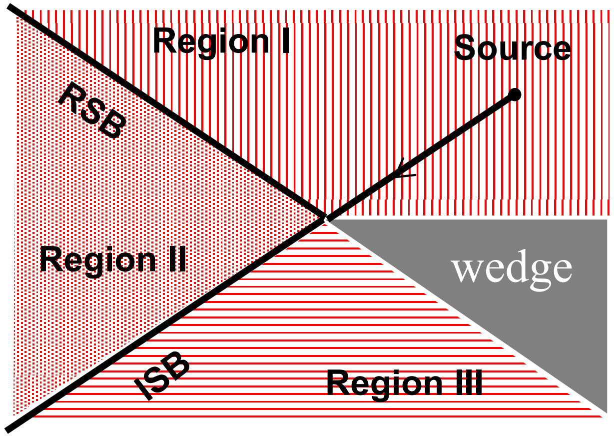
$$\boxed{A\left(\rho^d, s\right) = \sqrt{\frac{\rho^d}{s \cdot \left(\rho^d + s\right)}}}$$

For a straight edge we have:

$$A\left(\rho^d, s\right) = \begin{cases} \dfrac{1}{\sqrt{s}} & \text{for a plane incident wave} \\ \dfrac{1}{\sqrt{s \cdot \sin \beta_o'}} & \text{for a cylindrical incident wave} \\ \sqrt{\dfrac{\rho^d}{s \cdot \left(\rho^d + s\right)}} & \text{for a spherical incident wave} \end{cases}$$

- For the computation of the diffraction coefficients we refer in the following to a simple case with a cylindrical incident wave.

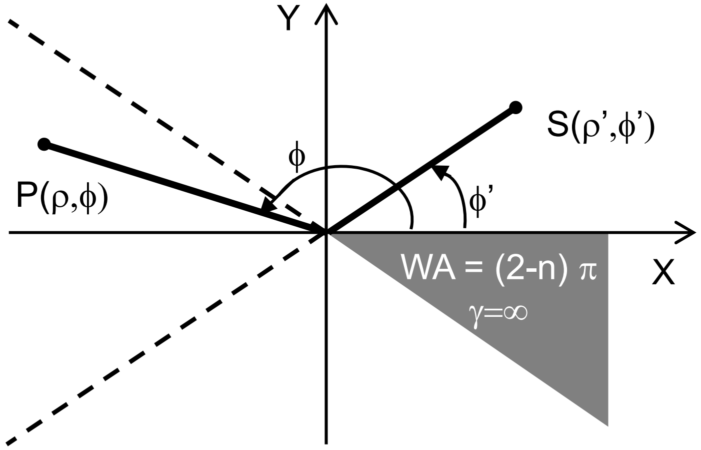
# The diffraction coefficients for a canonical 2D problem



ISB : Incidence Shadow Boundary

RSB : Reflection Shadow Boundary

R I : direct + reflected + diffracted
R II : direct + diffracted
R III : diffracted



Hypotheses:

- unlimited perfectly conducting wedge of angular width WA =$(2-n)\pi$ $(0 \leq n < 2)$
- Infinite uniform linear source parallel to the edge with constant current $I_0\,\mathbf{i_z}$

⇩

cylindrical incident wave with normal incidence

# The diffraction coefficients

Adopting the method described above the following Keller’s diffraction coefficients are obtained (***G**eometrical **T**heory of **D**iffraction, GTD*) [9]

$$D^S(\phi,\phi',n)=\frac{-e^{-j\pi/4}\cdot\sin\left(\pi/n\right)}{n\sqrt{2\pi\beta}}\cdot\left[\frac{1}{\cos\left(\pi/n\right)-\cos\left(\xi^-/n\right)}-\frac{1}{\cos\left(\pi/n\right)-\cos\left(\xi^+/n\right)}\right]$$

$$D^H(\phi,\phi',n)=\frac{-e^{-j\pi/4}\cdot\sin\left(\pi/n\right)}{n\sqrt{2\pi\beta}}\cdot\left[\frac{1}{\cos\left(\pi/n\right)-\cos\left(\xi^-/n\right)}+\frac{1}{\cos\left(\pi/n\right)-\cos\left(\xi^+/n\right)}\right]$$

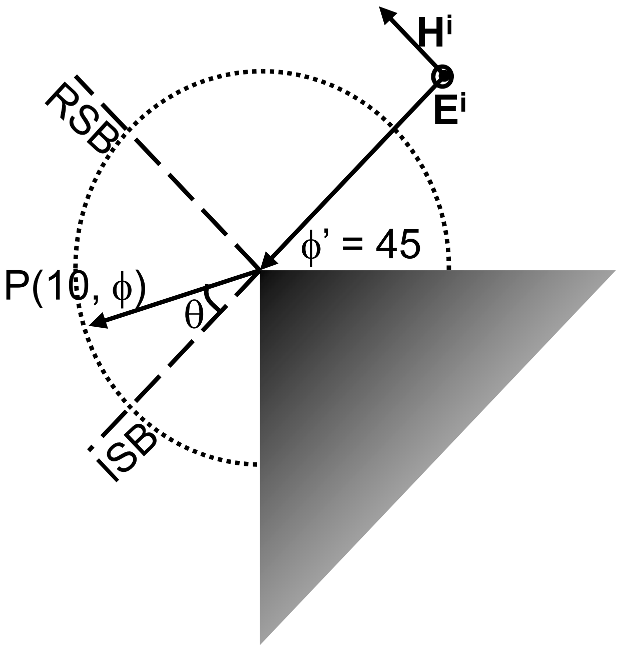
$\xi^- = \phi - \phi'$
$\xi^+ = \phi + \phi'$

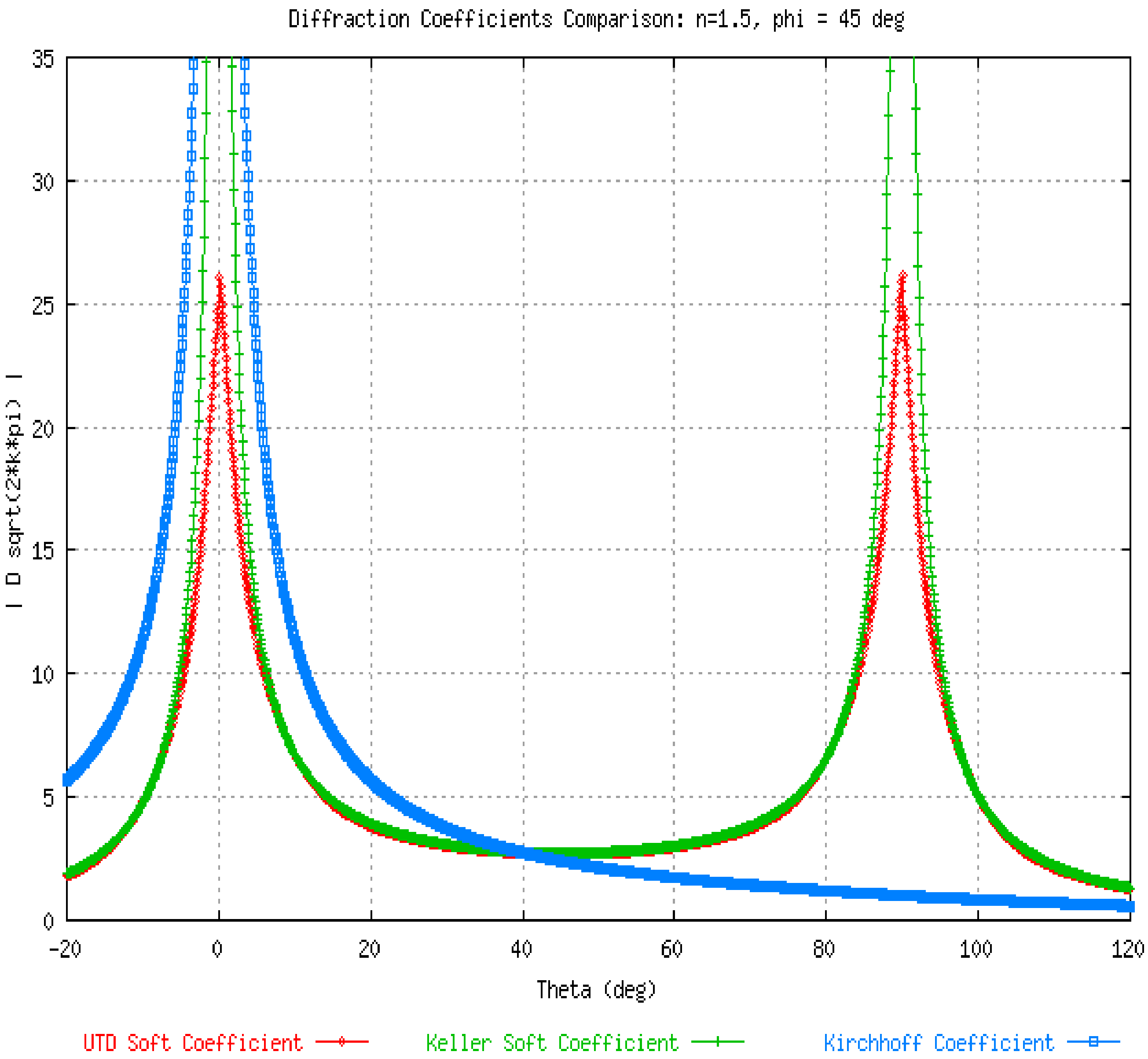
Such coefficients have singularities on the shadow boundaries, i.e. when:

$\xi- = \phi - \phi' = \pi$ (ISB)
$\xi+ = \phi + \phi' = \pi$ (RSB)

Therefore also other, more complicated coefficients have been derived which do not have such singularity: the UTD (***U**niform **T**heory of **D**iffraction*) coefficients

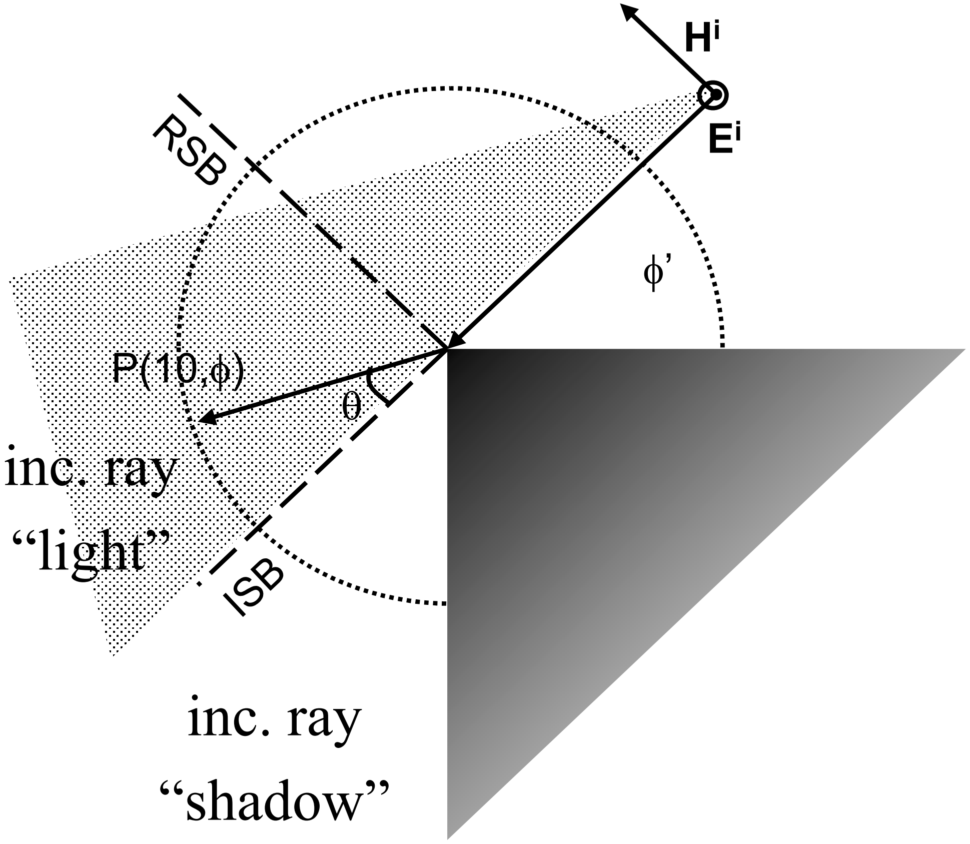
# Example (1/2)

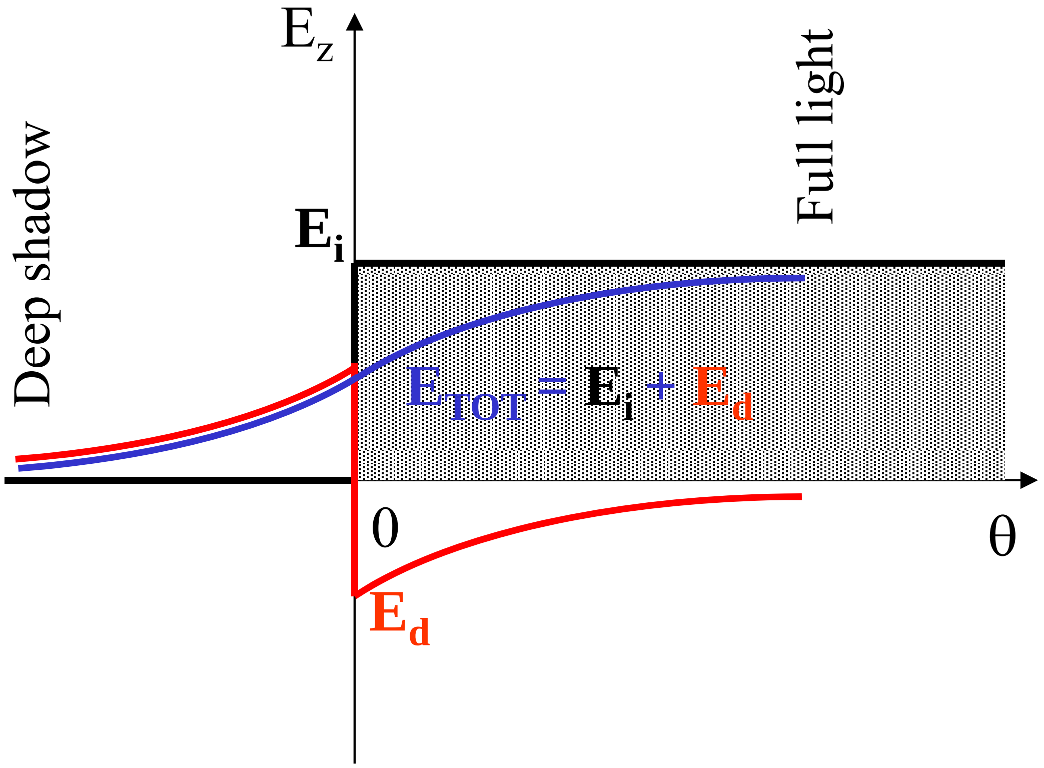

$H^i$
$E^i$
RSB
ISB
ϕ’ = 45
P(10, ϕ)
θ


Diffraction Coefficients Comparison: n=1.5, phi = 45 deg
| D sqrt(2*k*pi) |
35
30
25
20
15
10
5
0
-20
0
20
40
60
80
100
120
Theta (deg)
UTD Soft Coefficient
Keller Soft Coefficient
Kirchhoff Coefficient

# Example (2/2)

UTD, considering the diffracted ray and the incident ray





# Other notes on GTP

- A single ray can undergo multiple interactions. The resulting ray is therefore a polygonal line and the proper interaction coefficients must be applied for each interaction. The proper divergence factor must then be applied for the overall piece-wise path.

- Reflection and transmission does not change the form of the divergence factor of a ray. Diffraction does.

- Diffraction coefficients for oblique incident and dielectric wedges have also been derived by some authors

- The interaction called “diffuse scattering” is important but is not treated here. It will be briefly treated further on.

# Computation Examples: reflection

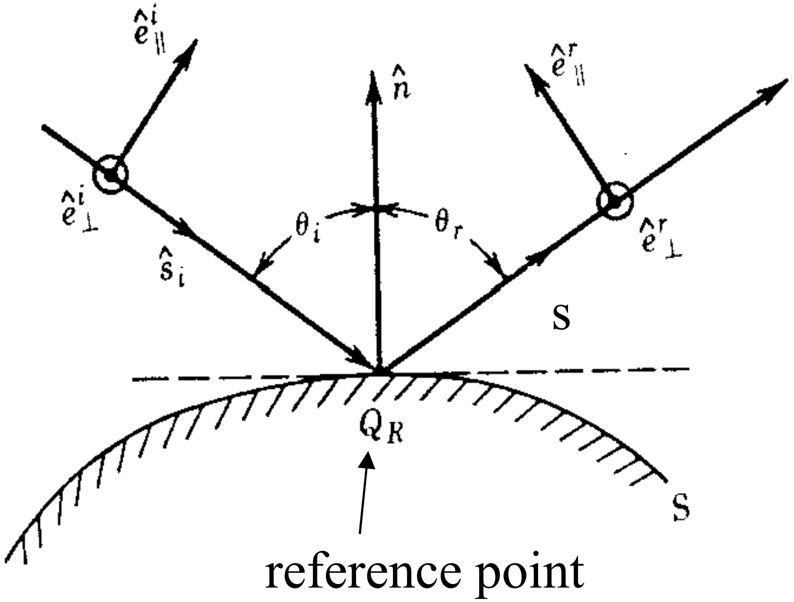
For the generic incident astigmatic wave we can write:

$$\vec{E}_r(s) = \underbrace{\vec{E}(Q_R)}_{\substack{\text{field at reference}\\ \text{point }(Q_R\text{ , }s=0)}} \cdot \underbrace{\underline{\underline{\mathbf{R}}}(Q_R,\theta_i)}_{\substack{\text{Reflection coefficient}\\ \text{(Dyadic)}}} \cdot \underbrace{\sqrt{\frac{\rho_1\cdot\rho_2}{(\rho_1+s)\cdot(\rho_2+s)}}}_{\substack{\text{divergence or spreading}\\ \text{factor}}} \cdot \underbrace{e^{-j\beta s}}_{\text{Phase factor}}$$

The use of the Dyadic Reflection coefficient [8] allows to refer to a fixed reference system

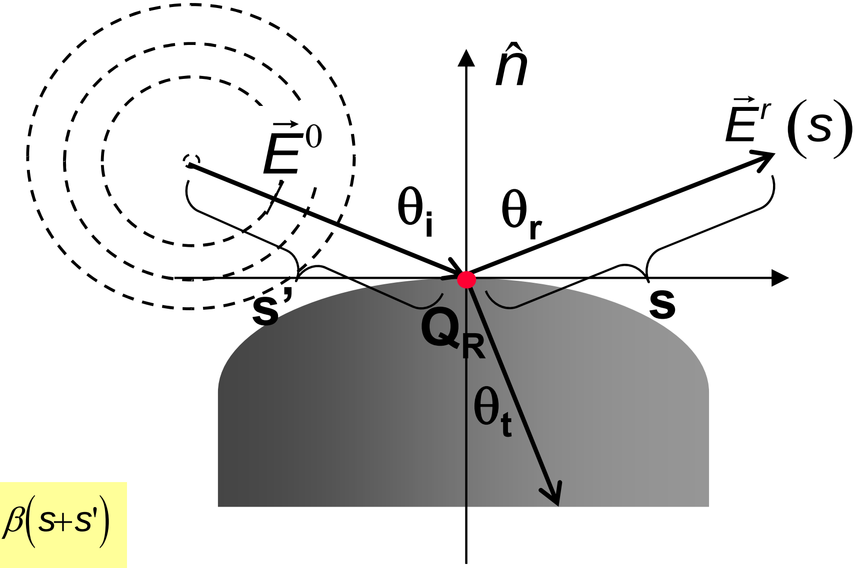
$$\underline{\underline{\mathbf{R}}} = \Gamma_{//}\left(\hat{e}^{i}_{//}\hat{e}^{r}_{//}\right) + \Gamma_{\perp}\left(\hat{e}^{i}_{\perp}\hat{e}^{r}_{\perp}\right)$$

$$\left(\bar{a}\,\bar{b}\right) \triangleq \begin{pmatrix} a_x b_x & a_x b_y & a_x b_z \\ a_y b_x & a_y b_y & a_y b_z \\ a_z b_x & a_z b_y & a_z b_z \end{pmatrix}$$



# Reflection (II)

For a spherical incident wave the expression above becomes ($\rho_1=\rho_2 = s'$):



$$\vec{E}_r(s) = \vec{E}^0 \frac{e^{-j\beta s'}}{s'} \cdot \underline{\underline{\mathbf{R}}} \cdot \frac{s'}{s+s'} e^{-j\beta s} = \vec{E}^0 \cdot \underline{\underline{\mathbf{R}}} \cdot \frac{e^{-j\beta(s+s')}}{s+s'}$$

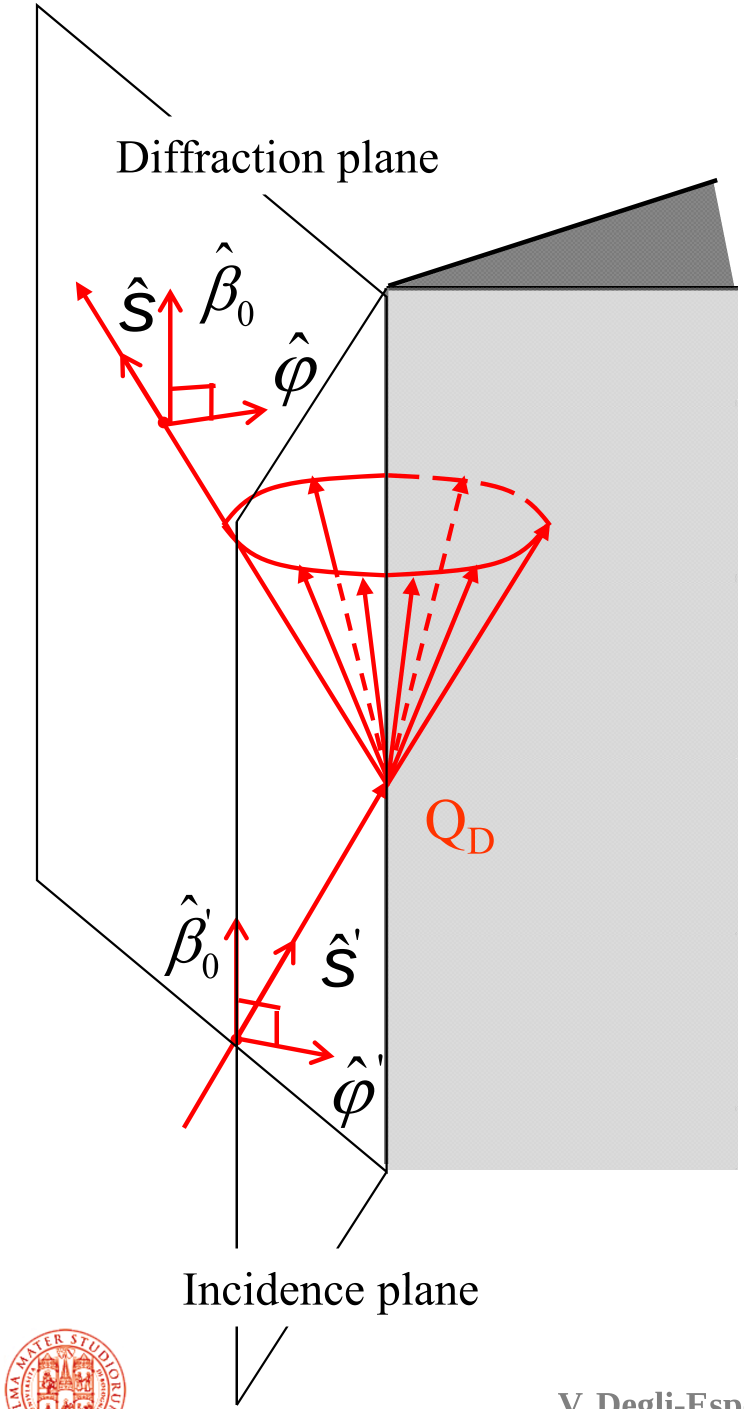
which is equivalent to

$$\begin{bmatrix} \vec{E}_{r\ TE}(s) \\ \vec{E}_{r\ TM}(s) \end{bmatrix} = \begin{bmatrix} \Gamma_{TE} & 0 \\ 0 & \Gamma_{TM} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}^0_{TE} \\ \vec{E}^0_{TM} \end{bmatrix} \cdot \frac{e^{-j\beta s'}}{s'} \frac{s'}{s+s'} e^{-j\beta s}$$

Divergence factor for a spherical wave

Incident field in $Q_R$

# Diffraction



Diffraction coefficients ➔ Diffracted field

$$\begin{bmatrix} E^d_{\beta_0} \\ E^d_{\phi} \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & D_h \end{bmatrix} \begin{bmatrix} E^i_{\beta'_0}\left(Q_D\right) \\ E^i_{\phi'}\left(Q_D\right) \end{bmatrix} \cdot A \cdot e^{-j\beta s}$$

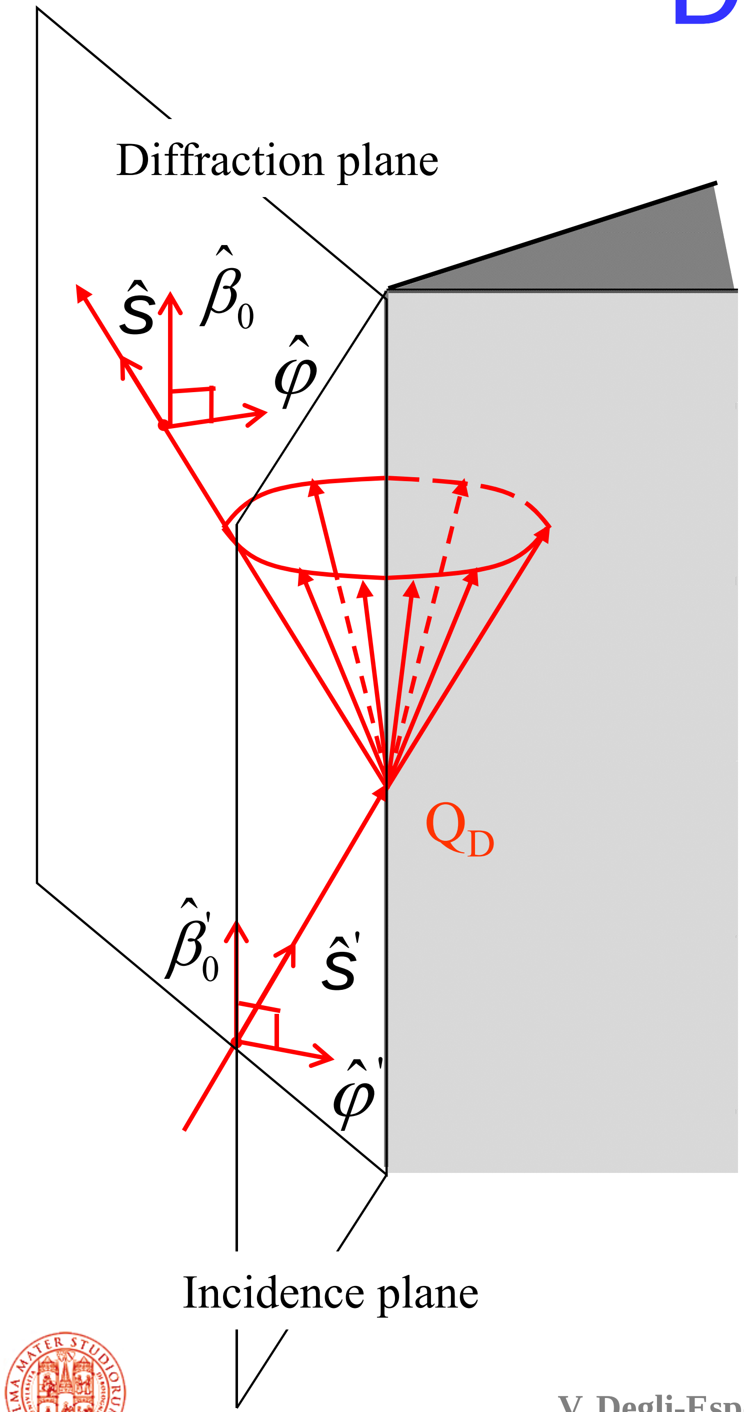
A is the *divergence factor* for the diffracted field. For a spherical incident wave:

$$A\left(s', s\right) = \sqrt{\frac{s'}{s \cdot \left(s' + s\right)}} \qquad \vec{E}^i\left(Q_D\right) = \vec{E}^{0i}\,\frac{e^{-j\beta s'}}{s'}$$

Therefore we have:

$$\begin{bmatrix} \vec{E}^d_{\beta_0} \\ \vec{E}^d_{\phi} \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & D_h \end{bmatrix} \begin{bmatrix} \vec{E}^{0i}_{\beta'_0} \\ \vec{E}^{0i}_{\phi'} \end{bmatrix} \cdot \frac{1}{\sqrt{s \cdot s' \cdot \left(s' + s\right)}} \cdot e^{-j\beta\left(s+s'\right)}$$

# Diffraction (II)



Using the the <u>Dyadic Diffraction coefficient</u>:

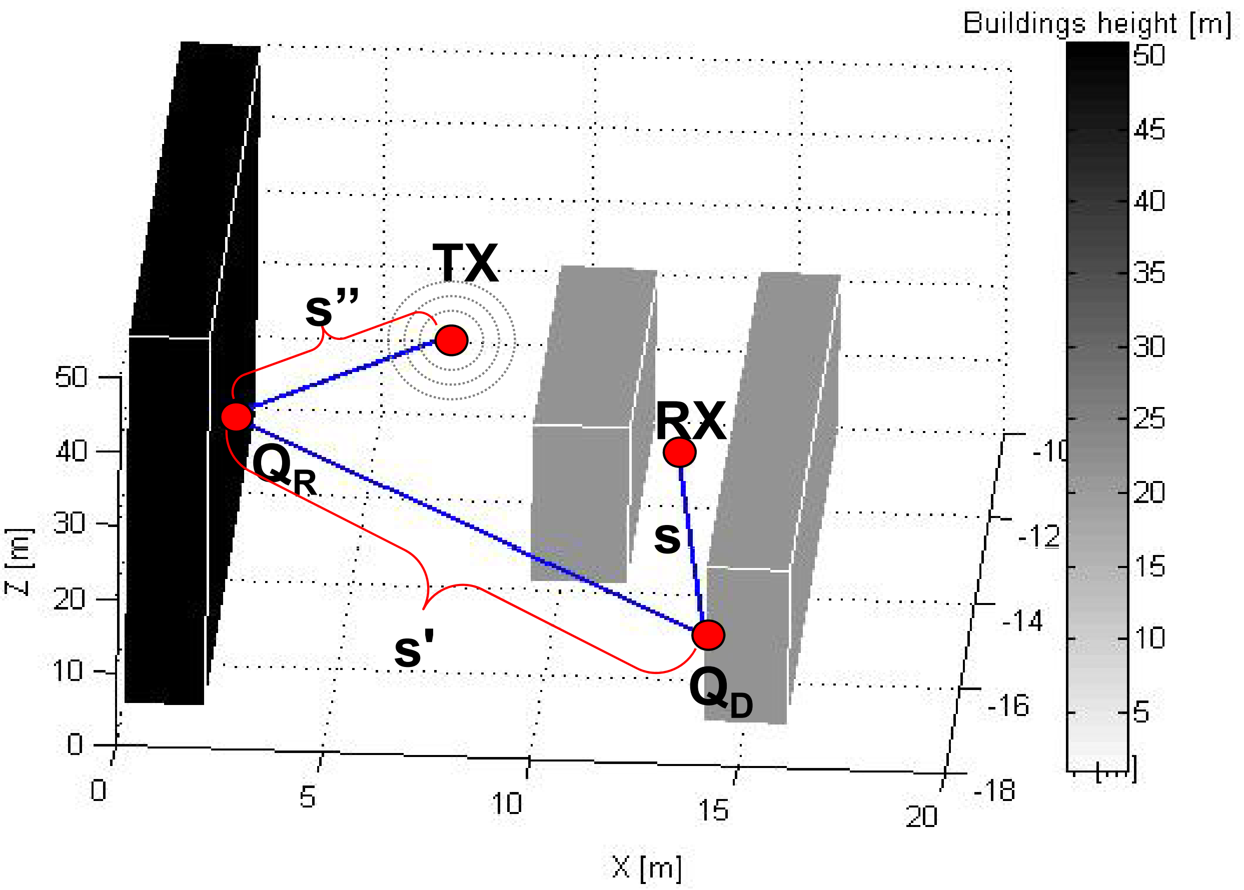
$$\underline{\underline{\mathbf{D}}} = D_s\left(\hat{\beta}_0'\hat{\beta}_0\right) + D_h\left(\hat{\phi}'\hat{\phi}\right)$$

we have

$$\overline{E}^d = \overline{E}^0 \cdot \underline{\underline{\mathbf{D}}} \cdot \frac{1}{\sqrt{s \cdot s' \cdot \left(s' + s\right)}} \cdot e^{-j\beta\left(s+s'\right)}$$

# Double interaction (1/2)

## Reflection + Vertical Edge Diffraction



Field at the reflection point: $\vec{E}(Q_R) = \vec{E}^0 \dfrac{e^{-j\beta s''}}{s''}$

# Double interaction (2/2)

The field at the diffraction point is:

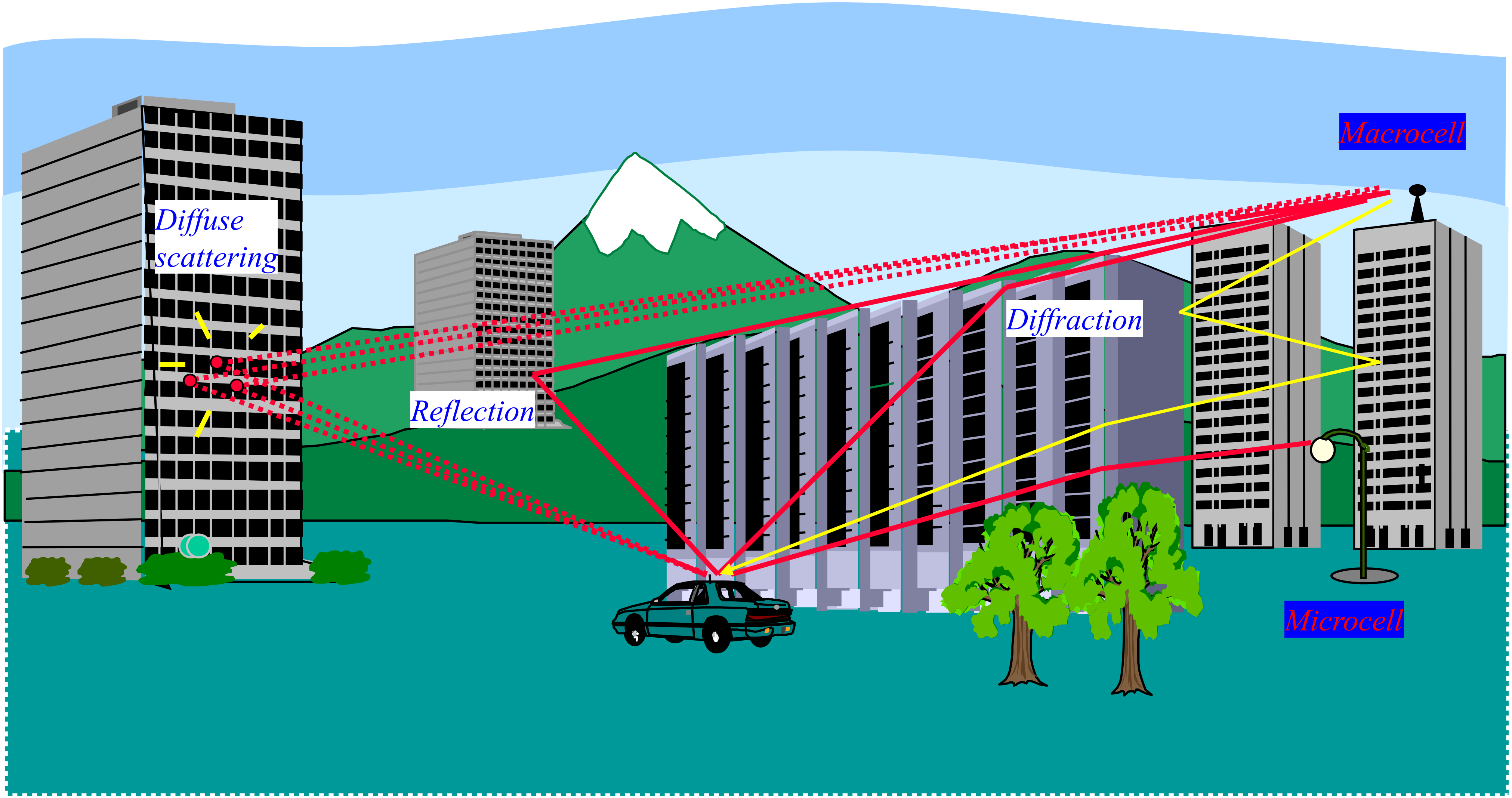
$$\vec{E}(Q_D) = \underbrace{\vec{E}^0 \cdot \frac{e^{-j\beta s''}}{s''}}_{\vec{E}(Q_R)} \cdot \underline{\underline{\mathbf{R}}} \cdot \frac{s''}{s'+s''} e^{-j\beta s'} = \vec{E}^0 \cdot \underline{\underline{\mathbf{R}}} \cdot \frac{e^{-j\beta(s'+s'')}}{s'+s''}$$

Finally, the field at the RX can be computed as:

$$\boxed{\begin{aligned} \vec{E}(Rx) &= \vec{E}(Q_D) \cdot \underline{\underline{\mathbf{D}}} \cdot \sqrt{\frac{(s'+s'')}{s\left[s+(s'+s'')\right]}} \cdot e^{-j\beta s} = \\ &= \vec{E}^0 \cdot \underline{\underline{\mathbf{R}}} \cdot \underline{\underline{\mathbf{D}}} \cdot \frac{1}{s'+s''} \cdot \sqrt{\frac{(s'+s'')}{s\left[s+(s'+s'')\right]}} \cdot e^{-j\beta(s+s'+s'')} = \\ &= \vec{E}^0 \cdot \underline{\underline{\mathbf{R}}} \cdot \underline{\underline{\mathbf{D}}} \cdot \frac{1}{\sqrt{s(s'+s'')(s+s'+s'')}} \cdot e^{-j\beta(s+s'+s'')} \end{aligned}}$$

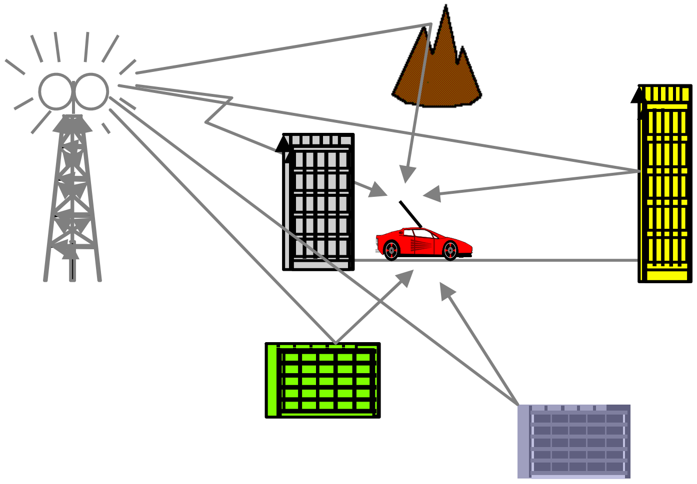
# Superposition of multiple rays (1/2)

## (Multipath propagation…)



# Superposition of multiple rays (2/2)

*Multipath propagation → not only attenuation !*



Multipath

Attenuation (fading)

Time dispersion

*Delay spread*

Angle/space dispersion

*Angle spread*

Some systems can *exploit* multipath, others only *cope with* it

# References

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